Exam #: ______________________

Printed Name: ______________________

Signature: ______________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Master's Final Examination and Ph.D. Qualifying Examination

PART II

Tuesday, March 29, 2011, 12:30 p.m. to 4:30 p.m.

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

You are encouraged to use the integrals, constants, and other information on the following two pages where appropriate to help you solve the problems.

There are eight equally weighted questions, each beginning on a new page. Read all eight questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. When you start a new problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic and will be provided. **Personal calculators are not allowed**. Dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries are not allowed. No other papers or books may be used.**

When you have finished, come to the front of the room, put all problems in numerical order and staple them together with this sheet on top. Then hand your examination paper to the proctor.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand your exam paper in on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg (0.511 MeV/c^2)
Proton rest mass (m_p)	1.673×10^{-27} kg (938 MeV/c^2)
Neutron rest mass (m_n)	1.675×10^{-27} kg (940 MeV/c^2)
Atomic mass unit (AMU)	1.7×10^{-27} kg
Atomic weight of a nitrogen atom	14 AMU
Atomic weight of an oxygen atom	16 AMU
Planck's constant (h)	6.63×10^{-34} J·s
Speed of light in vacuum (c)	3.00×10^{8} m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Gravitational constant (G)	6.67×10^{-11}N·m^2/kg^2
Permeability of free space (μ_0)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space (ϵ_0)	8.85×10^{-12} F/m
Mass of earth (M_E)	5.98×10^{24} kg
Equatorial radius of earth (R_E)	6.38×10^{6} m
Radius of sun (R_S)	6.96×10^{8} m
Classical electron radius (r_0)	2.82×10^{-15} m
Specific heat of oxygen (c_V)	21.1 J/mole·K
Specific heat of oxygen (c_P)	29.4 J/mole·K
Specific heat of water (0^o C $< T <$ 100^o C)	4.18 J/(g·K)
Latent heat, ice $\rightarrow$ water	334 J/g
Latent heat, water $\rightarrow$ steam	2257 J/g
Gravitational acceleration on Earth (g)	9.8 m/s^2
1 atmosphere	1.01×10^{5} Pa

Pauli spin matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \;,\; \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \;,\; \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \;. \qquad (1)$$

Some integrals

$$
\begin{aligned}
\int_{-\infty}^{+\infty} dx\; e^{-x^2} &= \sqrt{\pi} \\
\int d^3r\; e^{-i\vec{q}\cdot\vec{r}} A(r) &= \frac{4\pi}{q} \int_0^{\infty} r\,dr\; \sin(qr) A(r) \\
\int_0^{\infty} r\,dr\; \sin(qr)\, e^{-r^2/a^2} &= \frac{q a^3 \sqrt{\pi}}{4}\, e^{-q^2 a^2/4} \\
\int_0^{\infty} dx\; x^n e^{-ax} &= \frac{n!}{a^{n+1}} \\
\int_0^{\infty} u\,du\; \sin(bu)\, e^{-cu} &= \frac{2bc}{(b^2+c^2)^2} \\
\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \int_{-a}^{a} dz\; \Theta(x^2+y^2<a^2)\,(x^2+y^2) &= \pi\, a^5 \\
\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \int_{-a}^{a} dz\; \Theta(x^2+y^2<a^2) &= 2\pi\, a^3 \\
\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dz\; \Theta(x^2+y^2+z^2<a^2)\,(x^2+y^2) &= \frac{8\pi}{15}\, a^5 \\
\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dz\; \Theta(x^2+y^2+z^2<a^2) &= \frac{4\pi}{3}\, a^3
\end{aligned}
\tag{2}
$$

Trigonometric identities

$$
\begin{aligned}
1-\cos\theta &= 2\,\sin^2(\theta/2) \\
1+\cos\theta &= 2\,\cos^2(\theta/2) \\
\sin\theta &= 2\,\sin(\theta/2)\,\cos(\theta/2)
\end{aligned}
\tag{3}
$$

Problem 1

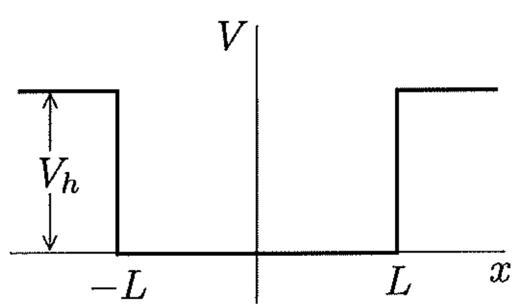
A particle of mass m in one dimension is subject to a potential $V(x)$ that vanishes in the region $-L < x < L$ and equals a positive constant, V_h, outside of this region. The particle is in an energy eigenstate with energy E. Consider the case that $E < V_h$. You may use units with $\hbar = 1$ if you like.

(a) If one does an experiment to measure the position the particle, would it be possible (in quantum mechanics) to find the particle in the region $x > L$? Explain briefly why or why not.

(b) What is the form of the Schrödinger equation in the region $x > L$?

(c) What is the form of the solution of the Schrödinger equation in the region $x > L$, given restrictions from physics considerations that supplement the differential equation? What physics considerations apply?

(d) Supposing that the state is the energy eigenstate with the lowest possible energy, sketch the solution showing what happens in the regions $x < L$, $-L < x < L$ and $L < x$ and at the boundaries between these regions. You do not need to find the solution for the lowest energy eigenstate; it is enough to make a sketch that shows the qualitative behavior.



Problem 2

Consider a free particle of mass m in one dimension. You may use units with $\hbar = 1$ if you like.

(a) Write down the time dependent Schrödinger equation for this situation and show that

$$\psi_k(x,t) = Ae^{i(kx-\omega t)} \tag{1}$$

satisfies the Schrödinger equation if ω is related to k in a certain way. Find ω as a function of k so that $\psi(x,t)$ does satisfy the Schrödinger equation.

(b) Suppose now that $\psi(x,t)$ obeys the free particle Schrödinger equation and that at time $t = 0$ it is

$$\psi(x,0) = \frac{2^{1/4}}{a^{1/2}\pi^{1/4}}\, e^{-x^2/a^2} \quad . \tag{2}$$

This function is normalizable, and has in fact been normalized so that

$$\int dx\ |\psi(x,0)|^2 = 1 \ . \tag{3}$$

Another way to write this is

$$\psi(x,0) = \frac{a^{1/2}}{2^{3/4}\pi^{3/4}} \int dk\ e^{ikx} e^{-a^2k^2/4} \quad . \tag{4}$$

Find $\psi(x,t)$ for $t > 0$.

Problem 3

An electron is confined to a cubical box of side length a. (That is, there is a potential that equals zero inside the box and is infinite outside the box.) The electron has spin $1/2$ and mass m_e. You may use units with $\hbar = 1$ if you like. However, part (e) asks for a numerical result in meters.

(a) What is the ground state energy E_0 of the electron in this box?

(b) What is the energy E_1 of the first excited state?

(c) What is the degeneracy of the first excited state? That is, how many quantum states are there with the energy E_1?

(d) If the electron is in one of the quantum states with energy E_1, the system can decay to the ground state by emitting a photon of wavelength λ. Please find λ in terms of a and m_e.

(e) Supposing that $a = 1$ nm, what is λ in units of meters?

Problem 4

Consider a particle of mass m that is elastically scattered according to non-relativistic quantum mechanics by a Gaussian potential in three dimensions,

$$V(\vec{r}) = -V_0 e^{-\vec{r}^2/a^2} \quad . \tag{1}$$

Here $V_0 > 0$ and a is a parameter with dimension of length. The incoming particle has wavevector $\vec{k}$ and, after the scattering, the outgoing particle has wavevector $\vec{k}'$. Define $\vec{q} = \vec{k}' - \vec{k}$ and denote the scattering angle by θ. You may use units with $\hbar = 1$ if you like.

(a) Show that $|\vec{q}| = 2|\vec{k}| \sin(\theta/2)$.

(b) Using the Born approximation, calculate the differential scattering cross section $d\sigma/d\Omega$.

(c) Making use of this result, calculate the total scattering cross section σ_T.

Problem 5

The cross section of a coaxial transmission line is shown in the Figure. It consists of a solid copper rod of radius R_1 that carries a uniform current I into the page, and a concentric copper tube that surrounds the rod and carries the uniform return current I out of the page. A non-magnetic insulator fills the space between the rod and the tube. The inner and outer radii of the copper tube are R_2 and R_3, respectively.

Write down the equation that connects $\vec{\nabla} \times \vec{B}$ with $\vec{j}$ (where $\vec{B}$ is the magnetic field and $\vec{j}$ is the electrical current density.)

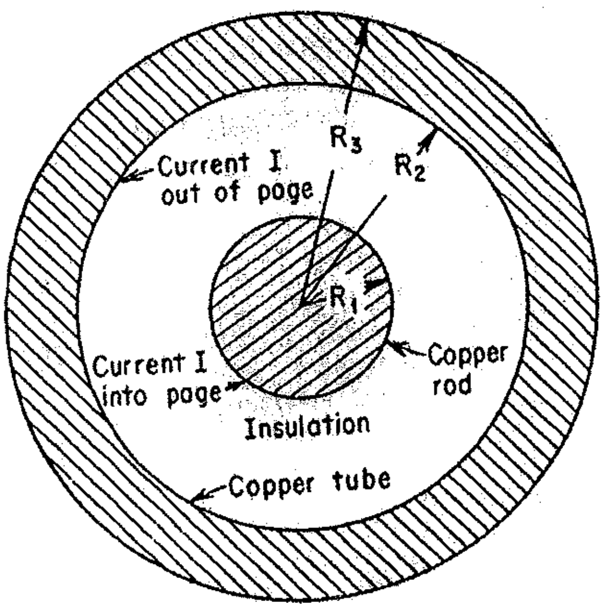
Use this equation to calculate the magnitude of the magnetic field, $B(r)$, as a function of r (where r is the distance from the axis of the cylinder) in the following four regions:

(a) $r < R_1$ (i.e. inside the copper rod)

(b) $R_1 < r < R_2$ (i.e. between the two conductors)

(c) $R_2 < r < R_3$ (i.e. inside the copper tube)

(d) $r > R_3$ (i.e. outside the coaxial line)



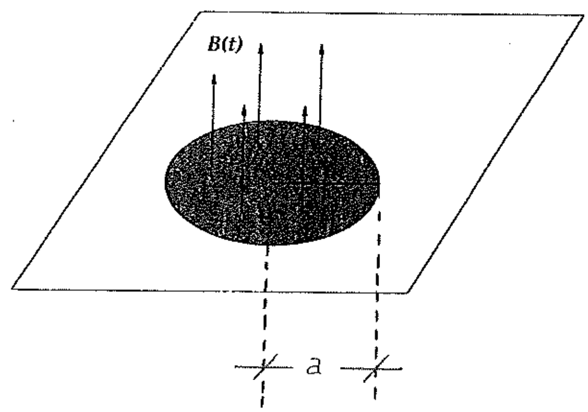
Problem 6

Consider a disk of radius a in a plane. Let the disk be penetrated by a homogeneous magnetic field $\vec{B}$. Let the direction of $\vec{B}$ be perpendicular to the plane, and let its modulus be B.

(a) Suppose $\vec{B}$ is time independent. Show that there can not be any closed electric field loops in the plane.

(b) Suppose that $\vec{B} = \vec{B}(t)$ is time dependent. Show that there is an electric field in the plane and determine its magnitude on the perimeter of the disk.

(c) If $\vec{B}$ points upward (see Figure) and is increasing with time, and the disk is made of copper, in which direction does the induced electric current flow? (Draw it as seen from above the plane of the disk.)



Problem 7

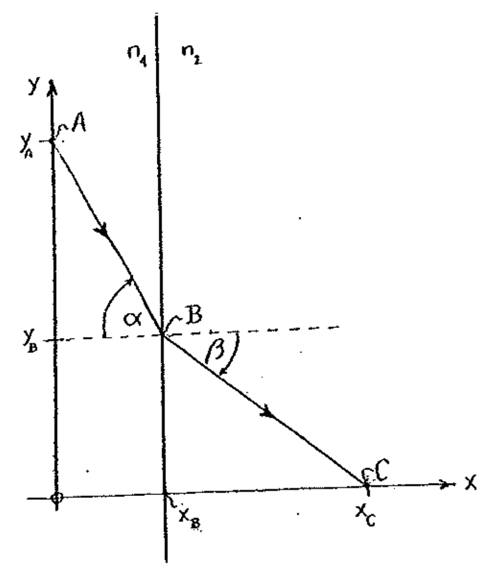
Ray optics can be formulated as a variational principle: the actual path taken by an optical ray through an optical medium extremizes the optical path length.

Consider the transmission of a light ray across the plane boundary between two uniform media. (See Figure). The index of refraction in the first and second medium is n_1 and n_2, respectively. The ray starts at point A, crosses the boundary at some point B (not given), and ends at point C.

Derive the relationship between the angles α and β, when the optical path length of the beam between A and C is extremum.

You may assume that in a uniform medium the shortest line connecting two points is a straight line.

(Hint: It may help to express the path length in terms of angles.)



Problem 8

A transatlantic cable can be modeled as an infinitely long circuit of the form shown in the Figure, where R_1 and R_2 are resistances and ... denotes an infinite repetition of the same sequence.

(a) Calculate the resistance of this circuit between the points a and b. (Hint: What is the resistance between the points c and d when the circuit is cut along the dashed line?)

(b) Evaluate the result when $R_1 \ll R_2$.