Exam #: ______________________

Printed Name: ______________________

Signature: ______________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Master's Final Examination and Ph.D. Qualifying Examination

PART I

Monday, March 28, 2011, 12:30 p.m. to 4:30 p.m.

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

You are encouraged to use the integrals, constants, and other information on the following two pages where appropriate to help you solve the problems.

There are eight equally weighted questions, each beginning on a new page. Read all eight questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. When you start a new problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic and will be provided. **Personal calculators are not allowed.** Dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries are not allowed. No other papers or books may be used.**

When you have finished, come to the front of the room, put all problems in numerical order and staple them together with this sheet on top. Then hand your examination paper to the proctor.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand your exam paper in on time, an appropriate number of points may be subtracted from your final score.

## Constants

| | |
|---|---|
| Electron charge ($e$) | $1.60 \times 10^{-19}$ C |
| Electron rest mass ($m_e$) | $9.11 \times 10^{-31}$ kg ($0.511$ MeV/c$^2$) |
| Proton rest mass ($m_p$) | $1.673 \times 10^{-27}$ kg ($938$ MeV/c$^2$) |
| Neutron rest mass ($m_n$) | $1.675 \times 10^{-27}$ kg ($940$ MeV/c$^2$) |
| Atomic mass unit (AMU) | $1.7 \times 10^{-27}$ kg |
| Atomic weight of a nitrogen atom | 14 AMU |
| Atomic weight of an oxygen atom | 16 AMU |
| Planck's constant ($h$) | $6.63 \times 10^{-34}$ J·s |
| Speed of light in vacuum ($c$) | $3.00 \times 10^{8}$ m/s |
| Boltzmann's constant ($k_B$) | $1.38 \times 10^{-23}$ J/K |
| Gravitational constant ($G$) | $6.67 \times 10^{-11}$N·m$^2$/kg$^2$ |
| Permeability of free space ($\mu_0$) | $4\pi \times 10^{-7}$ H/m |
| Permittivity of free space ($\epsilon_0$) | $8.85 \times 10^{-12}$ F/m |
| Mass of earth ($M_E$) | $5.98 \times 10^{24}$ kg |
| Equatorial radius of earth ($R_E$) | $6.38 \times 10^{6}$ m |
| Radius of sun ($R_S$) | $6.96 \times 10^{8}$ m |
| Classical electron radius ($r_0$) | $2.82 \times 10^{-15}$ m |
| Specific heat of oxygen ($c_V$) | 21.1 J/mole·K |
| Specific heat of oxygen ($c_P$) | 29.4 J/mole·K |
| Specific heat of water ($0^o\,\mathrm{C} < T < 100^o\,\mathrm{C}$) | 4.18 J/(g·K) |
| Latent heat, ice $\rightarrow$ water | 334 J/g |
| Latent heat, water $\rightarrow$ steam | 2257 J/g |
| Gravitational acceleration on Earth ($g$) | 9.8 m/s$^2$ |
| 1 atmosphere | $1.01 \times 10^{5}$ Pa |

## Pauli spin matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \ , \ \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \ , \ \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \ . \tag{1}$$

**Some integrals**

$$
\begin{aligned}
\int_{-\infty}^{+\infty} dx\; e^{-x^2} &= \sqrt{\pi} \\
\int d^3r\; e^{-i\vec{q}\cdot\vec{r}} A(r) &= \frac{4\pi}{q} \int_0^\infty r\,dr\; \sin(qr) A(r) \\
\int_0^\infty r\,dr\; \sin(qr)\, e^{-r^2/a^2} &= \frac{qa^3\sqrt{\pi}}{4}\, e^{-q^2a^2/4} \\
\int_0^\infty dx\; x^n e^{-ax} &= \frac{n!}{a^{n+1}} \\
\int_0^\infty u\,du\; \sin(bu)\, e^{-cu} &= \frac{2bc}{(b^2+c^2)^2} \qquad (2) \\
\int_{-\infty}^\infty dx \int_{-\infty}^\infty dy \int_{-a}^{a} dz\; \Theta(x^2+y^2<a^2)\,(x^2+y^2) &= \pi\, a^5 \\
\int_{-\infty}^\infty dx \int_{-\infty}^\infty dy \int_{-a}^{a} dz\; \Theta(x^2+y^2<a^2) &= 2\pi\, a^3 \\
\int_{-\infty}^\infty dx \int_{-\infty}^\infty dy \int_{-\infty}^\infty dz\; \Theta(x^2+y^2+z^2<a^2)\,(x^2+y^2) &= \frac{8\pi}{15}\, a^5 \\
\int_{-\infty}^\infty dx \int_{-\infty}^\infty dy \int_{-\infty}^\infty dz\; \Theta(x^2+y^2+z^2<a^2) &= \frac{4\pi}{3}\, a^3
\end{aligned}
$$

**Trigonometric identities**

$$
\begin{aligned}
1-\cos\theta &= 2\,\sin^2(\theta/2) \\
1+\cos\theta &= 2\,\cos^2(\theta/2) \qquad (3) \\
\sin\theta &= 2\,\sin(\theta/2)\,\cos(\theta/2)
\end{aligned}
$$

**Problem 1**

In a classical model of a molecule, a charge $-q$ of mass $m$ is attached to another charge $q$ with infinite mass by a harmonic "spring" with spring constant $C$. The motion of the negative charge is damped, with the damping force proportional to the velocity of the charge and the damping rate (imaginary frequency) given by $\gamma$.

a) (4 points) When the molecule is driven by monochromatic light with frequency $\omega$ and electric field $E_0 e^{i\omega t}$, polarized along the axis of the molecule, determine the steady-state amplitude of the molecular vibration induced by the light.

b) (4 points) Determine the electric susceptibility (ratio of induced dipole moment to applied field) for the molecule near resonance (including only contributions related to the spring).

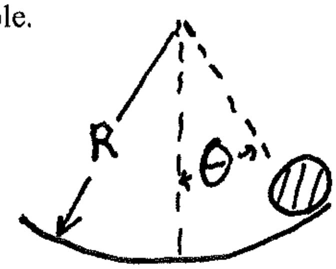
c) (2 points) Using the above result, plot schematically both the real and imaginary parts of the susceptibility near the resonance. Give a brief description of the physical meaning of each part.

**Problem 2**

A small solid marble with radius $r$ and mass $m$ rolls without slipping on the interior surface of a hollow cylinder with interior radius $R$, in the plane of the Figure.

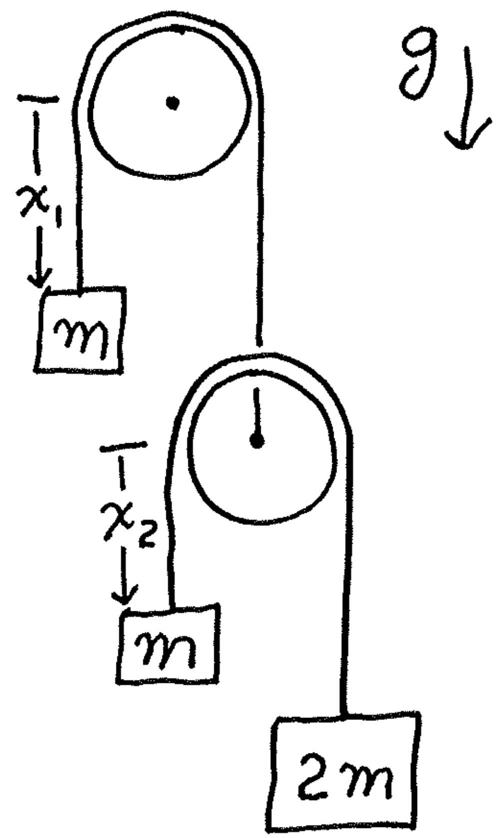
a) Write down the kinetic energy of the marble in terms of $d\theta/dt$ (see Fig.).

b) For small $\theta$, find the oscillation frequency of the marble.

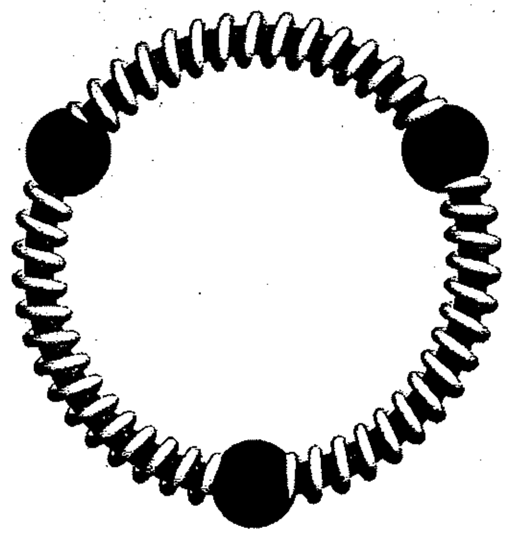


**Problem 3**

A compound Atwood machine is shown in the Figure. The top pulley is fixed, and the lower pulley is attached to the cable. The pulleys and cables may be taken as frictionless and massless, and the other masses are indicated in the Figure. Write down the Lagrangian for the system in terms of $x_1$ and $x_2$ as in the Figure. Write down the Euler-Lagrange equations for the motion of this system. Solve these equations to obtain the accelerations of the masses relative to the fixed pulley (be sure to indicate the direction of each acceleration).



**Problem 4**



Three identical point masses (mass $= m$) are constrained to move on a frictionless, horizontal hoop. Three identical springs (spring constant $= k$) connect the masses and wrap around the hoop as shown in the Figure. Find the normal modes and their respective frequencies.

## Problem 5

Consider a classical gas consisting of $N$ spinless point particles in a three-dimensional volume $V$. Assume the limit of massless particles traveling at the speed of light, so that the relation between the momentum $\vec{p}_i$ of particle $i$ and its energy $E_i$ becomes $E_i = c\,|\vec{p}_i|$. Thus, the energy of a gas of $N$ of these particles is $c\sum_{i=1}^{N} |\vec{p}_i|$ for all particle positions inside volume $V$.

(a) Calculate the canonical partition function of this gas.

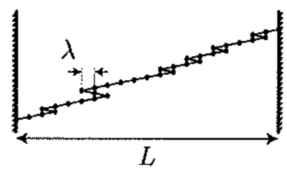
(b) Calculate the free energy of the gas as a function of its natural variables: temperature $T$, volume $V$, and number of particles $N$.

(c) Derive the equation of state (i.e., the relation between pressure $P$, volume $V$, temperature $T$, and number of particles $N$) of the gas.

(d) Express the internal energy $U$ of the gas in terms of the temperature $T$ and the number of particles $N$.

## Problem 6

Consider a one-dimensional chain of $N$ identical links, each of length $\lambda$, attached between two walls as shown, as a statistical "paper-clip" model of rubber elasticity. Each link can point exactly forward or exactly backward. Let $L$ denote the overall length of the (possibly folded) chain. The *tension* $F$ of the chain is the inward force exerted by the chain on the walls. The total energy $U$ and the number $N$ are kept constant in this problem. The entropy $S$ and the length $L$ are extensive variables, with corresponding intensive variables $T$ and $F$, respectively.




(a) Write down a general expression for the tension on the ends of the chain in terms of $S$ at constant $U$ and $N$.

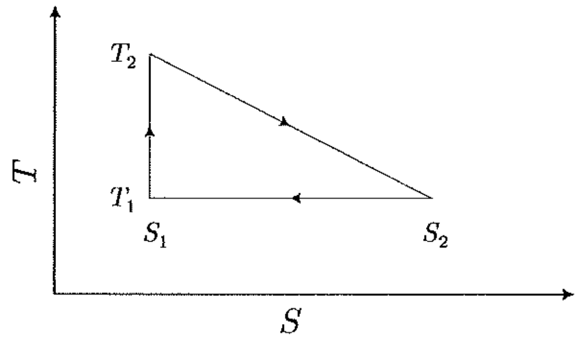
(b) Let $N_r$ and $N_\ell$ denote the number of links pointing to the right or left, respectively, with $N_r + N_\ell = N$. Derive an expression for the (microcanonical) entropy in terms of $N$ and $N_r$ (i.e., eliminate $N_\ell$), in the limit where $N$, $N_r$, and $N_\ell$ are large enough that you may use Stirling's approximation $\log(N!) \approx N \log N - N$, and analogously for $N_r$, $N_\ell$. Also ignore any "edge effects," treating all links as statistically equivalent and mutually independent.

(c) Derive an expression for the tension on the chain in terms of $N$ and $L$, eliminating $N_r$.

(d) Finally, give the lowest-order expression for the force in the limit $N\lambda \gg L$. Give a brief physical interpretation of your result.

## Problem 7

Consider a cyclic (reversible) heat engine that operates as in the diagram.




(a) What is the efficiency of this engine?

(b) What does the corresponding diagram for a Carnot engine look like in the $T$–$S$ plane?

## Problem 8

Consider a system of $N = 2$ spinless, noninteracting bosons with two non-degenerate single-particle energy levels $\varepsilon_0 = 0$ and $\varepsilon_1 \equiv \varepsilon > 0$. The particle number $N$ is fixed.

(a) Make a table of all possible states of the system. Each state is identified by the pair of single-particle occupation numbers $(n_0, n_1)$ with $n_0 + n_1 = N$, and has a total energy $E_{(n_0,n_1)}$.

(b) Write down the canonical partition function of the system at temperature $T$.

(c) Find the average number $\langle n_0 \rangle$ of particles in the ground state at temperature $T$.

(d) What will $\langle n_0 \rangle$ be in the limits $T \to 0$ and $T \to \infty$?