Exam #: ______________________

Printed Name: ______________________

Signature: ______________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Master's Final Examination and Ph.D. Qualifying Examination, Part II

Tuesday, 30 March 2010, 9:00 a.m. to 1:00 p.m.

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are eight equally weighted questions, each beginning on a new page. Read all eight questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic, and will be provided. **Personal calculators of any type are not allowed.** Paper dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries will not be allowed. No other papers or books may be used.**

When you have finished, come to the front of the room and hand your examination paper to the proctor; first put all problems in numerical order and staple them together.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand in your exam paper on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg (0.511 MeV/c^2)
Proton rest mass (m_p)	1.673×10^{-27} kg (938 MeV/c^2)
Neutron rest mass (m_n)	1.675×10^{-27} kg (940 MeV/c^2)
W^- rest mass (m_W)	80.4 GeV/c^2
Planck's constant (h)	6.63×10^{-34} J $\cdot$ s
Speed of light in vacuum (c)	3.00×10^{8} m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Heat Capacity of Water	4.19 J/K/cm^3
Gravitational constant (G)	6.67×10^{-11}N $\cdot$ m^2/kg^2
Permeability of free space (μ_0)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space (ϵ_0)	8.85×10^{-12} F/m
Mass of Earth ($M_\oplus$)	5.98×10^{24} kg
Mass of Moon (M_{Moon})	7.35×10^{22} kg
Mass of Sun ($M_\odot$)	1.99×10^{30} kg
Radius of Earth ($R_\oplus$)	6.38×10^{6} m
Radius of Moon (M_{Moon})	1.74×10^{6} m
Radius of Sun ($R_\odot$)	6.96×10^{8} m
Earth - Sun distance ($R_{\oplus,\odot}$)	1.50×10^{11} m
Classical electron radius ($r_\circ$)	2.82×10^{-15} m
Gravitational acceleration on Earth (g)	9.8 m/s^2
Atomic mass unit	1.66×10^{-27} kg
One atmosphere (1 atm)	1.01×10^{5} N/m^2

Stirling's Approximation

$$\log N! \;\approx\; N \log N - N$$

Useful Definitions, Commutators, and Expectation Values

$$J_\pm = J_x \pm iJ_-$$

$$[J_i, J_k]_- = i\epsilon_{ikl} J_l$$

$$< j, m-1 \mid J_- \mid j, m >= \hbar[(j+m)(j-m+1)]^{1/2}$$

$$< j, m+1 \mid J_+ \mid j, m >= \hbar[(j-m)(j+m+1)]^{1/2}$$

Problem 1

A Hermitean operator A has eigenstates Ψ_a, with corresponding eigenvalues a. Another operator B has a commutator relation with A given by $[A, B] = \lambda B$, where λ is real. The system is prepared in the state $\Psi' = B\Psi_a$, and then the value of operator A is measured in this state. Predict the expectation value and the variance of this measurement.

Problem 2

Alice and Bob cooperate to prepare a state of two spin 1 particles in a state with total angular momentum zero. (Note that the particles have spin 1, not spin 1/2.) Then, without disturbing its spin, Alice takes one of the particles to her laboratory and measures the z-component of its spin angular momentum, finding a value $m_A \hbar$. Without disturbing its spin, Bob takes the other particle to his laboratory and measures the z-component of its spin angular momentum, finding a value $m_B \hbar$.

a. What are the values of m_A that Alice may find?

b. What are the values of m_B that Bob may find?

c. For each pair of values m_A, m_B, evaluate the probability $P(m_A, m_B)$ that Alice and Bob will find this pair of values.

Problem 3

For a three-dimensional isotropic harmonic oscillator with frequency ω, the energy states for a single particle can be written in terms of three non-negative integers n_x, n_y, n_z, as $E = (n_x + n_y + n_z + 3/2\,)\hbar\omega$.

a. Suppose we put 10 identical noninteracting fermions with spin-1/2 in such a harmonic oscillator potential. What is the lowest energy that this system can have?

b. What values can be measured for the total spin S (of all 10 spins together) in the lowest energy state discussed in part (a.) ? (There is more than one possibility.)

c. Now suppose we add a pairwise *tiny attractive* force between all the fermions. Argue whether now the lowest energy state will have a little lower or a little higher energy than was calculated in part (a). What will the total spin S be in the new lowest energy state? (There is a unique answer now.)

Problem 4

a. The ionization energy of a hydrogen atom in its ground state is $E = (1/2)\mu c^2\alpha^2$, where μ is the reduced mass of the electron proton system, c is the speed of light, and α is the fine structure constant. Calculate the value in electron volts of the ionization energy of hydrogen in its ground state.

b. A bound system of an electron and an anti-electron (positron) is referred to as positronium. Use your knowledge of the ionization energy of hydrogen, and the masses of the electron and the proton to estimate the ionization energy of positronium in its ground state. Express your answer in electron volts.

c. Positronium is formed by stopping anti-electrons in matter. It is found that the bound system is formed in two distinct states, both of which have orbital angular momentum $L = 0$. Consider the possible spin configurations of this system, to determine the expected total angular momentum for these two positronium states.

d. One of these positronium states decays preferentially to two photons, and the other decays preferentially to three photons. Which angular momentum state decays preferentially to two photons and which decays to three photons? Explain your answer.

Problem 5

A particle in thermal equilibrium with a heat bath at temperature T can be in any one of three states, whose energies are $-\epsilon_0$, 0 , and ϵ_0.

a. Calculate the mean energy $< E >$ energy of this system.

b. What are the limiting values of $< E >$ as $T \to 0$ and $T \to \infty$? How could you have guessed these answers?

c. Calculate the specific heat of this system as a function of temperature.

Problem 6

Give numerical values for the entropy change when:

a. Two thermally isolated containers of water, each of volume 1 cm^3, initially at temperatures of $295K$ and $305K$, respectively, are brought into thermal contact and allowed to come into thermal equilibrium.

b. A system of 10^{22} non-interacting spins, each of which can point either up or down, are completely aligned in magnetic field at low temperature. The magnetic field is then slowly reduced to zero, so that the orientation of each spin becomes completely random.

Problem 7

In the process of taking a gas from state a to state c along the curved path shown in the Figure below, $80J$ of heat leave the system and $55J$ of work are done *on* the system.

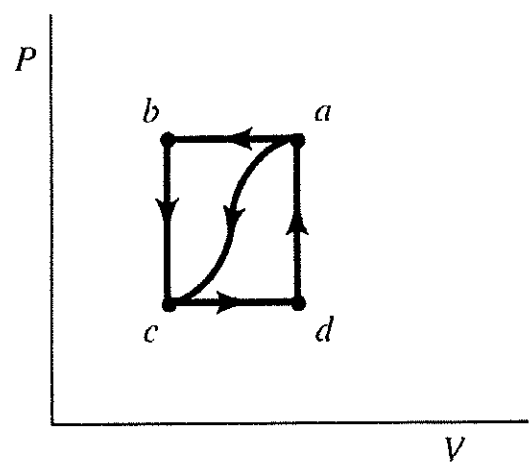
a. Determine the difference in internal energy $U_a - U_b$.

b. When the gas is taken along the path cda shown in the Figure, the work W done *by* the gas is $W = 38J$. How much heat Q is *added to* the gas in the process cda?

c. If $P_a = 2.5P_d$, how much work is done in the process abc?

d. What is Q for the path abc?

e. If $U_a - U_b = 10J$, what is Q for the process bc?



Problem 8

Consider the Earth to be in radiative equilibrium at a temperature T_E, receiving and re-radiating energy from the sun. The sun has temperature T_s, and radius R_s, and is a distance D_{se} from the Earth. Model the Earth's atmosphere as transparent to *all* of the sun's radiation, and absorbing a fraction x of the Earth's radiation. The separation between the atmosphere and the Earth's surface is negligible. Derive an expression for x in terms of T_E, T_s , R_s, and D_{se}. Treat the sun, Earth, and the Earth's atmosphere as black bodies.