Exam #: ______________________

Printed Name: ______________________

Signature: ______________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Master's Final Examination and Ph.D. Qualifying Examination, Part I

Monday, 29 March 2010, 9:00 a.m. to 1:00 p.m.

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are eight equally weighted questions, each beginning on a new page. Read all eight questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic, and will be provided. **Personal calculators of any type are not allowed**. Paper dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries will not be allowed. No other papers or books may be used.**

When you have finished, come to the front of the room and hand your examination paper to the proctor; first put all problems in numerical order and staple them together.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand in your exam paper on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	$1.60 \times 10^{-19}\,\mathrm{C}$
Electron rest mass (m_e)	$9.11 \times 10^{-31}\,\mathrm{kg}$ ($0.511\,\mathrm{MeV/c^2}$)
Proton rest mass (m_p)	$1.673 \times 10^{-27}\,\mathrm{kg}$ ($938\,\mathrm{MeV/c^2}$)
Neutron rest mass (m_n)	$1.675 \times 10^{-27}\,\mathrm{kg}$ ($940\,\mathrm{MeV/c^2}$)
W^- rest mass (m_W)	$80.4\,\mathrm{GeV/c^2}$
Planck's constant (h)	$6.63 \times 10^{-34}\,\mathrm{J \cdot s}$
Speed of light in vacuum (c)	$3.00 \times 10^{8}\,\mathrm{m/s}$
Boltzmann's constant (k_B)	$1.38 \times 10^{-23}\,\mathrm{J/K}$
Heat Capacity of Water	$4.19\ \mathrm{J/K/cm^3}$
Gravitational constant (G)	$6.67 \times 10^{-11}\mathrm{N \cdot m^2/kg^2}$
Permeability of free space (μ_0)	$4\pi \times 10^{-7}\,\mathrm{H/m}$
Permittivity of free space (ϵ_0)	$8.85 \times 10^{-12}\,\mathrm{F/m}$
Mass of Earth ($M_\oplus$)	$5.98 \times 10^{24}\,\mathrm{kg}$
Mass of Moon (M_{Moon})	$7.35 \times 10^{22}\,\mathrm{kg}$
Mass of Sun ($M_\odot$)	$1.99 \times 10^{30}\,\mathrm{kg}$
Radius of Earth ($R_\oplus$)	$6.38 \times 10^{6}\,\mathrm{m}$
Radius of Moon (M_{Moon})	$1.74 \times 10^{6}\,\mathrm{m}$
Radius of Sun ($R_\odot$)	$6.96 \times 10^{8}\,\mathrm{m}$
Earth - Sun distance ($R_{\oplus,\odot}$)	$1.50 \times 10^{11}\,\mathrm{m}$
Classical electron radius (r_o)	$2.82 \times 10^{-15}\,\mathrm{m}$
Gravitational acceleration on Earth (g)	$9.8\,\mathrm{m/s^2}$
Atomic mass unit	$1.66 \times 10^{-27}\,\mathrm{kg}$
One atmosphere (1 atm)	$1.01 \times 10^{5}\,\mathrm{N/m^2}$

Laplacian

$$\nabla^2\psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}$$

Divergence

$$\vec{\nabla}\cdot\vec{A} = \frac{1}{s}\frac{\partial}{\partial s}sA_s + \frac{1}{s}\frac{\partial}{\partial\phi}A_\phi + \frac{\partial}{\partial z}A_z$$

Vector Identities

$$\vec{\nabla}\times\left(\vec{\nabla}\times\vec{A}\right) = \vec{\nabla}\left(\vec{\nabla}\cdot\vec{A}\right) - \nabla^2\vec{A}$$

Legendre Polynomials

$$\begin{aligned} P_0(\theta) &= 1 \\ P_1(\theta) &= \cos\theta \\ P_2(\theta) &= \frac{1}{2}\left(3\cos^2\theta - 1\right) \end{aligned}$$

Problem 1

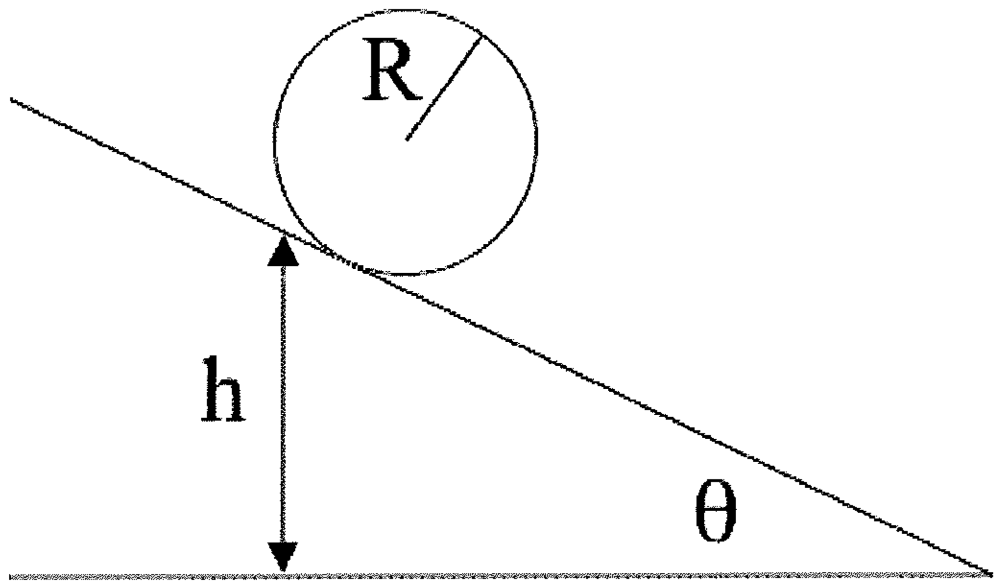
A passenger train is moving at 100 km/hr from Eugene to Portland (due north, latitude 45°)

a) Find the direction and magnitude of the Coriolis force on a pendulum bob of mass 100g, suspended by a string from the ceiling of the railway car.

b) Ignoring possible centrifugal forces, what is the angle made by the string with respect to the vertical?

Problem 2

A circular hoop of mass M and radius R rolls down a plane inclined at an angle θ in the Earth's gravitational field. For simplicity, assume the hoop is infinitely thin (i.e., all the mass is at distance R from the geometric center of the hoop), and rolls without slipping (see figure below).



Answer the following questions if the hoop starts at rest at height h:

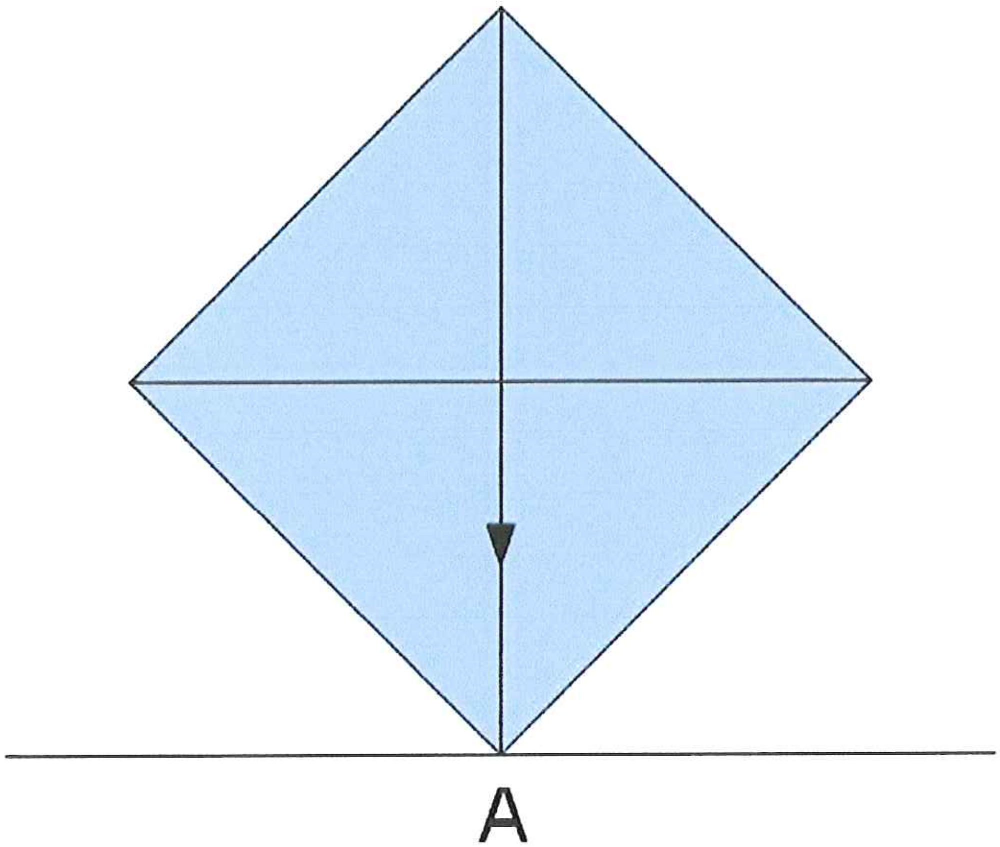
a) How fast is it moving when it reaches the bottom of the ramp?

b) What is the hoop's angular velocity at the bottom of the ramp?

c) How long does it take to reach the bottom of the ramp?

Problem 3

A cube with uniform density and side length, l, balances on one edge, and then falls. Determine its angular velocity when its face hits the ground. The cube does not slide; in other words, A is fixed. The center of mass is at the center.

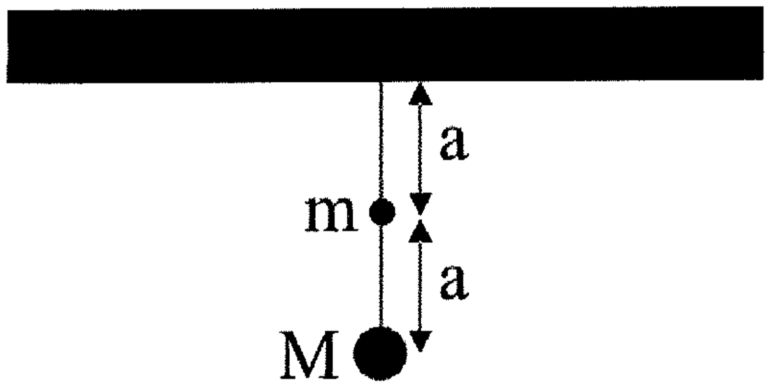


a) Given that the moment of inertia of the cube about A is I, how fast is the cube moving when it hits the table? Express your answer in terms of I, m, g and l where m is the mass of the cube and g is the gravitational acceleration.

b) Calculate I about A. Use I to calculate v.

Problem 4

A double pendulum consists of masses m and M attached to two nearly massless rods separated by a distance a as shown, such that motion occurs only in the plane of the page.



Considering small amplitude motion:

a) Write down the Lagrangian of the system;

b) Write the equations of motion; and

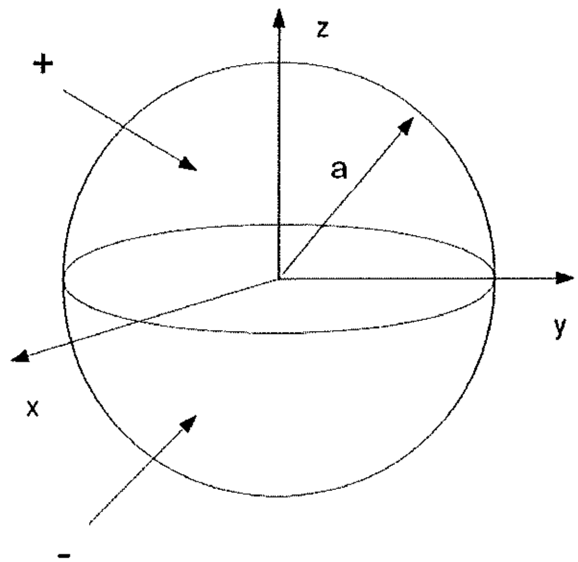
c) Calculate the frequencies of the normal modes of the system.

Problem 5

A sphere of radius a carries uniform charge density $\rho_\circ$ in one hemisphere and the opposite charge density $-\rho_\circ$ in the other hemisphere. The charged sphere produces an electric potential $\Phi(r, \theta, \phi)$, where r is the radial coordinate, θ is the polar angle, and ϕ is the azimuthal angle. For $r \gg a$, the potential to leading order is of the form

$$\Phi(r, \theta, \phi) \approx \frac{g_n(\theta, \phi)}{r^n}, \tag{1}$$

where n is a constant. Find $g_n(\theta, \phi)$ and the numerical value for n for $r \gg a$. Express $g_n(\theta, \phi)$ in terms of the angular coordinates, $\rho_\circ$, a, and fundamental constants.



Problem 6

Consider an isotropic medium with constant conductivity σ. There is no free charge present, that is, $\rho = 0$.

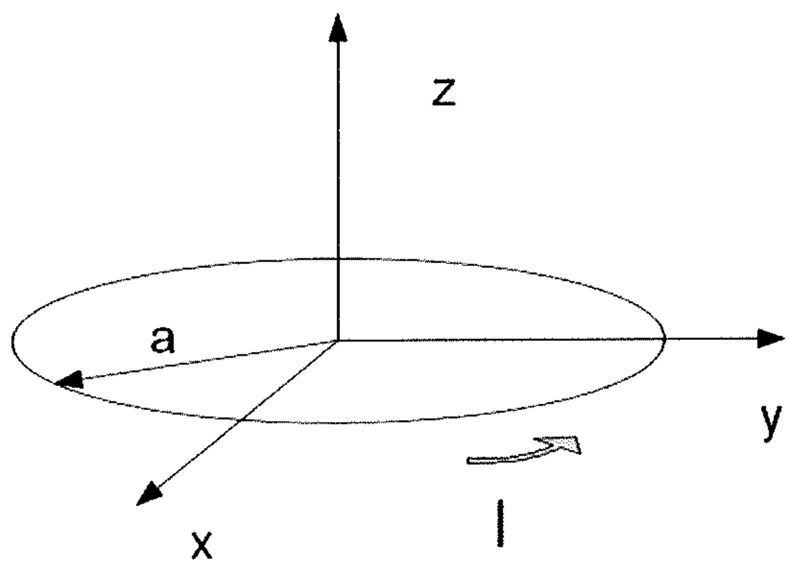
a. Write down the appropriate Maxwell equations for the medium.

b. Derive the damped wave equation for the electric field $\vec{E}$ in the medium. Assume that Ohm's law in the form $\vec{J} = \sigma \vec{E}$, where $\vec{J}$ is the current density and $\vec{E}$ is the electric field, is valid.

c. Show that monochromatic plane waves with frequency ω, damp exponentially with distance traveled in the medium. What is the decay length for the field *intensity* I in the medium?

Problem 7

Using the Biot-Savart Law, find the magnetic field $\vec{B}$ on the axis of a circular wire loop (radius a) which carries current I. Next, using $\vec{\nabla} \cdot \vec{B} = 0$, find an approximate expression for the radial component of the magnetic field, $B_s(s, \phi, z)$, at points slightly off-axis of the wire loop. Here s, ϕ, and z are the radial, azimuthal, and vertical cylindrical polar coordinates.



Problem 8

A capacitor is composed of two concentric spherical conducting shells with radii R and $2R$, which are maintained at potential difference V_o. The region enclosed between the two shells is empty between $0 < \theta < \pi/2$ and filled with dielectric material in the region $\pi/2 < \theta < \pi$. Here, θ is the polar angle. The material has dielectric constant $k = \epsilon/\epsilon_o$, where ϵ is the permittivity and ϵ_o is the permittivity of free space.

a. Explain why the electric field will be spherically symmetric (and in the radial direction) in the region between the two conducting shells.

b. Find the free charge surface density on the inner and outer shells of the capacitor in both the vacuum region and the region filled with dielectric.

c. Find the capacitance for this system.