Exam #: ____________________

Printed Name: _______________

Signature:__________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Ph.D. Qualifying Examination, PART II

Tuesday, March 30, 2004, 1:00 p.m. to 5:00 p.m.

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are six equally weighted questions, each beginning on a new page. Read all six questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start a new page.

If you need to leave the seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic. Calculators with stored equations or text are not allowed. Dictionaries may be used if they have been approved by the proctor before the examination begins. No other papers or books may be used.

When you have finished, come to the front of the room and hand your examination paper to the proctor; first put all problems in numerical order and staple them together.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand your exam paper in on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg (0.511 MeV/c^2)
Proton rest mass (m_p)	1.673×10^{-27} kg (938 MeV/c^2)
Neutron rest mass (m_n)	1.675×10^{-27} kg (940 MeV/c^2)
W+ rest mass (m_W)	80.4 Gev/c^2
Planck's constant (h)	6.63×10^{-34} J·s
Speed of light in vacuum (c)	3.00×10^{8} m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Gravitational constant (G)	6.67×10^{-11} Nm^2/kg^2
Permeability of free space (μ_0)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space (ε_0)	8.85×10^{-12} F/m
Mass of earth (M_E)	5.98×10^{24} kg
Equatorial radius of earth (R_E)	6.38×10^{6} m
Radius of sun (R_S)	6.96×10^{8} m
Temperature of surface of sun (T_S)	5.8×10^{3} K
Earth-sun distance (R_{ES})	1.5×10^{11} m
Density of iron at low temperature (ρ_{Fe})	7.88×10^{3} kg/m^3
Classical electron radius (r_0)	2.82×10^{-15} m
Gravitational acceleration on earth (g)	9.8 m/s^2
Atomic mass unit	$1.7\text{x}10^{-27}$ kg
Specific heat of oxygen (C_V)	21.1 J/mole·K
Specific heat of oxygen (C_P)	29.4 J/mole·K
Avogadro's number	$6.02\text{x}10^{23}$ atoms/mole
Gas constant (R)	8.31 J/mole·K

Stirling's formula:

$\ln(x!) \cong x\ln(x) - x - \ln(\sqrt{2\pi x})$ (the last term may often be neglected)

$$P_0(x) = 1$$

$$P_1(x) = x$$

$$P_2(x) = \frac{1}{2}\left(3x^2 - 1\right)$$

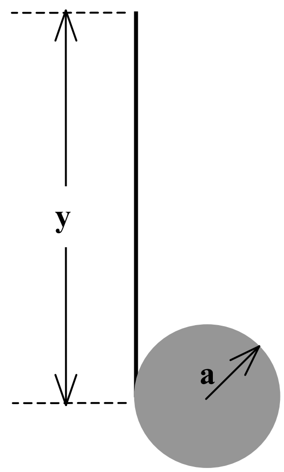
$$P_3(x) = \frac{1}{2}\left(5x^3 - 3x\right)$$

Problem 1

Consider an ideal yo-yo as shown below. The string is massless and the body of the yo-yo can be modeled as a uniform disk of radius a. Find the following when the yo-yo is released from rest:

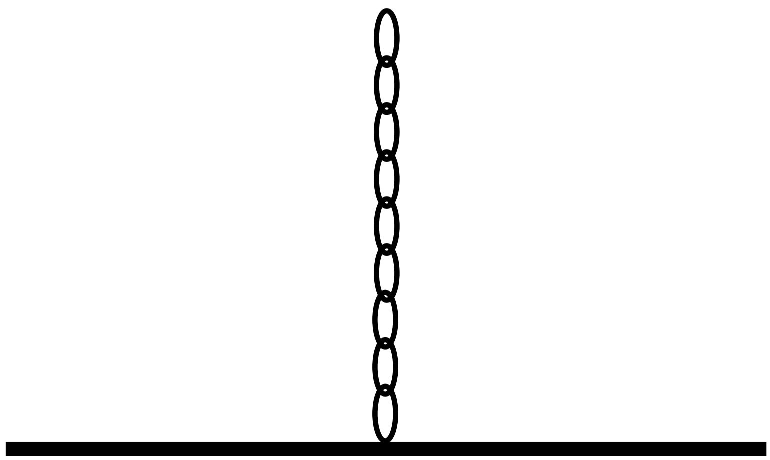
a) The velocity and position of the yo-yo as a function of time.

b) The tension in the string as a function of time.



Problem 2

A chain of mass M and length L is held vertically such that its lowest link is at the same level with the floor, as shown below. The chain is then released. Calculate the force exerted on the floor as a function of time. Assume the links are point masses and that they don't bounce when striking the floor.

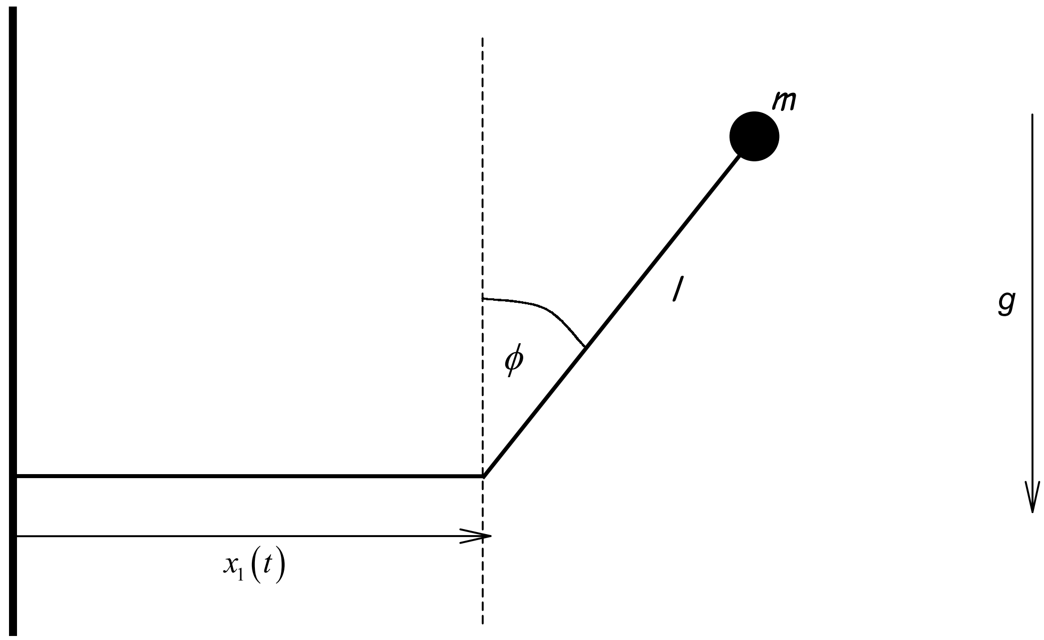


Problem 3

Consider an inverted pendulum whose base oscillates with frequency ω_0, amplitude a, and phase δ so that

$$x_1(t) = x_0 + a\cos(\omega_0 t + \delta).$$

Gravity points downwards.

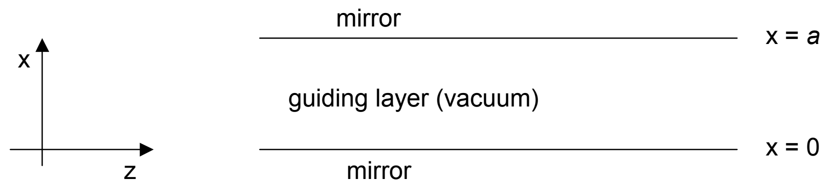


a) Write the Lagrangian of the system.

b) Find the equation of motion for ϕ. Show that this can be written as a driven linear harmonic oscillator for small ϕ.

Problem 4

A planar, optical wave guide is made with two parallel, perfectly reflecting mirrors, separated by a vacuum of thickness *a***.** The mirrors extend infinitely in the y-direction. The boundary condition is that the transverse field equals zero at the mirror surfaces. Take the x-coordinate so x = 0 at the lower mirror surface, and take the light to be traveling in z-direction.



a) Solve the wave equation in the guiding layer. Hint: try $\vec{E}(\vec{r}) = \vec{E}(x)\exp[i(kz - \omega t)]$ with the transverse component of $\vec{E}(x)$ proportional to sin(hx) or cos(hx), for some constant h. Write the resulting relation between h and k, where k is the z-component of the wave number.

b) By applying the above-mentioned boundary condition, find the allowed values of h_λ, where λ is the integer mode label.

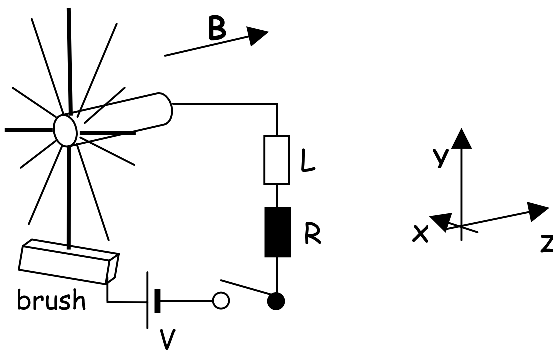
c) Find an expression for k_λ, corresponding to each guided mode.

d) Find an expression for the cut-off frequency of each mode.

e) Make a sketch of k_λ versus light frequency, showing several modes, and label the cut-offs.

Problem 5

The figure below shows an electric motor whose rate of rotation can be made to vary.



A metallic wheel consisting of a large number of thin spokes spanning the xy-plane can freely rotate about an axle parallel to the z-direction. The wheel has radius a and its moment of inertia with respect to the axle is J. A conducting brush always makes contact with one spoke at a time. An electric circuit is formed by the wheel and its axle, a battery supplying a voltage V, and wires connecting the brush, the battery, and the axle. The circuit has a resistance R and an inductance L. A homogeneous, time-independent magnetic field B is applied parallel to the axle. At time t = 0 the switch is closed, allowing a current I to flow. Mechanical friction is negligible.

(a) Show that the Lorentz force and the laws of mechanics lead to a linear relation between the current I(t) and the angular acceleration of the wheel, $\dot{\omega}(t)$.

(b) Use energy conservation to find a second relation between I(t) and $\omega(t)$. Combine this with the result of (a) to find a differential equation for the angular velocity of the wheel.

(c) What generic physical system has the same differential equation? Explain the physical meaning of the various terms in the equation you obtained in (b), and briefly describe the behavior of the system.

Problem 6

Two infinitely thin concentric spheres are located in vacuum and have radius R_1 and R_2, respectively, where $R_1 < R_2$. The electrostatic potential, Φ, on both spheres is symmetric around the z-axis.
As a function of polar angle θ (measured from the z-axis) the potential on the inner sphere is $V_0(3 + 2\cos(\theta))$, while on the outer sphere it is $(-2V_0)(3 + 2\cos(\theta))$.

Find the potential as a function of the spherical coordinates $\Phi(r, \theta, \varphi)$

a) inside the inner sphere,
b) in the space between the two spheres,
c) outside the larger sphere.
d) Write the potential between the two spheres in the limit $R_2 \rightarrow \infty$, while R_1 remains finite.
e) In cases (c) and (d) one deals with a free standing sphere in a surrounding infinite, empty space. Compare and comment on the results obtained in (c) and (d).