Exam #: ______________________________

Printed Name: ______________________________

Signature: ______________________________

PHYSICS DEPARTMENT
UNIVERSITY OF OREGON
Ph.D. Qualifying Examination, Part II
Tuesday, April 1, 2003, 1:00 p.m. to 5:00 p.m.

The examination pages are numbered in the upper left-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are six equally weighted questions, each beginning on a new page. Read all six questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic. **Calculators with stored equations or text are not allowed.** Dictionaries may be used if they have been approved by the proctor before the examination begins. **No other papers or books may be used.**

When you have finished the exam, come to the front of the room and hand your examination paper to the proctor; first put all problems in numerical order and staple them together.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand your exam paper in on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg (0.511 MeV/c^2)
Proton rest mass (m_p)	1.673×10^{-27} kg (938 MeV/c^2)
Neutron rest mass (m_n)	1.675×10^{-27} kg (940 MeV/c^2)
W^+ rest mass (m_W)	80.4 GeV/c^2
Planck's constant (h)	6.63×10^{-34} J-s
Speed of light in vacuum (c)	3.00×10^{8} m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Stefan-Boltzmann constant (σ)	5.67×10^{-8} J/(m^2-s-K^4)
Gravitational constant (G)	6.67×10^{-11} N-m^2/kg^2
Permeability of free space ($\mu_\circ$)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space ($\epsilon_\circ$)	8.85×10^{-12} F/m
Mass of Earth (M_E)	5.98×10^{24} kg
Equatorial radius of Earth (R_E)	6.38×10^{6} m
Radius of Sun (R_S)	6.96×10^{8} m
Mass of Sun (M_S)	1.99×10^{30} kg
Temperature of surface of the Sun (T_S)	$5,800$ K
Earth-Sun distance (R_{ES})	1.50×10^{11} m
Gravitational acceleration on Earth (g)	9.8 m/s^2
atomic mass unit	1.7×10^{-27} kg

Stirling's Formula

$$\ln(x!) = x\ \ln(x) - x - \ln\sqrt{2\pi x} + \mathcal{O}(1/x) \tag{1}$$

Larmor Radiation Formula

$$P = (\frac{2}{3c^3})q^2a^2 \tag{2}$$

Integrals

$$\int_{-\infty}^{\infty} dx\ x^{2n}\exp(-ax^2) = \frac{1 \times 3 \times 5 \times \cdots \times (2n-1)}{2^n a^n}\sqrt{\frac{\pi}{a}} \tag{3}$$

$$\int_0^{\infty} \frac{dx}{x}\ x^n \exp(-x) = \Gamma(n) \tag{4}$$

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln(x + \sqrt{x^2+a^2}) \tag{5}$$

$$\text{if Re(a)} > 0 \text{ and Im(b)} = 0 \text{ then } \int_{-\infty}^{\infty} e^{-ay^2} e^{iby} dy = e^{-\frac{b^2}{4a}}\sqrt{\frac{\pi}{a}} \tag{6}$$

Problem 1

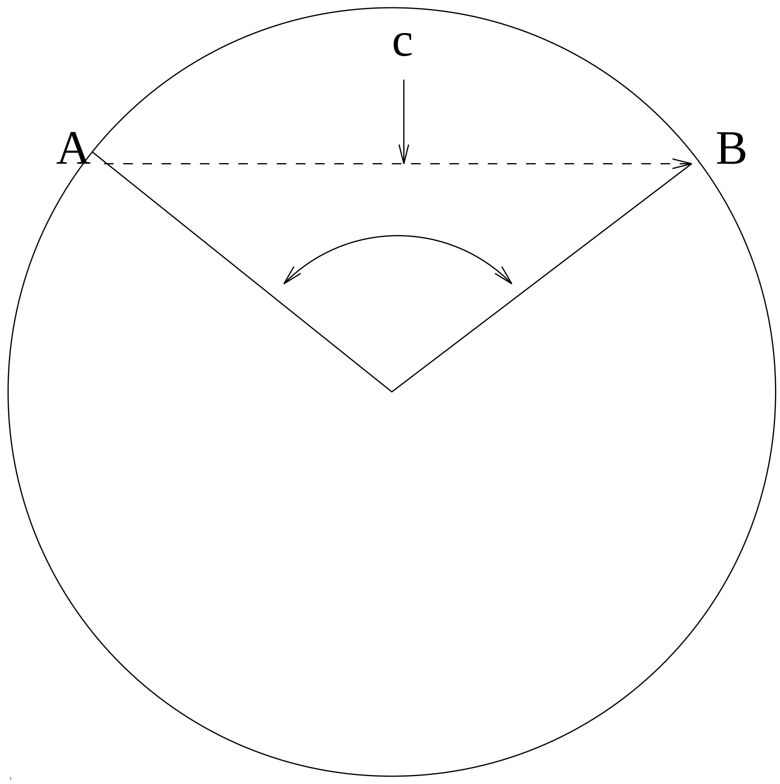
A mass m moves in a circular orbit of radius $r_\circ$ under the influence of a central force whose potential is

$$V(r) = \frac{mk_\circ}{r^n},$$

where $k_\circ$ is a constant. The constant $k_\circ < 0$ if $n > 0$ and $k_\circ > 0$ if $n < 0$. Show that the orbit is stable under small radial perturbations (that is, the mass will oscillate in radius about the circular orbit) if $n < 2$.

Problem 2

Consider a modern transportation system which uses a straight tunnel bored through the Earth to connect two points on the surface of the Earth (see figure). Ignore practical complications, such as friction, air resistance, increased temperature with depth, the Earth's liquid core, and the rotation of the Earth. Assume the Earth's density is uniform.



a. Calculate the time to fall freely from A to B and show that the time is independent of the angle ϕ. The particle starts from rest.

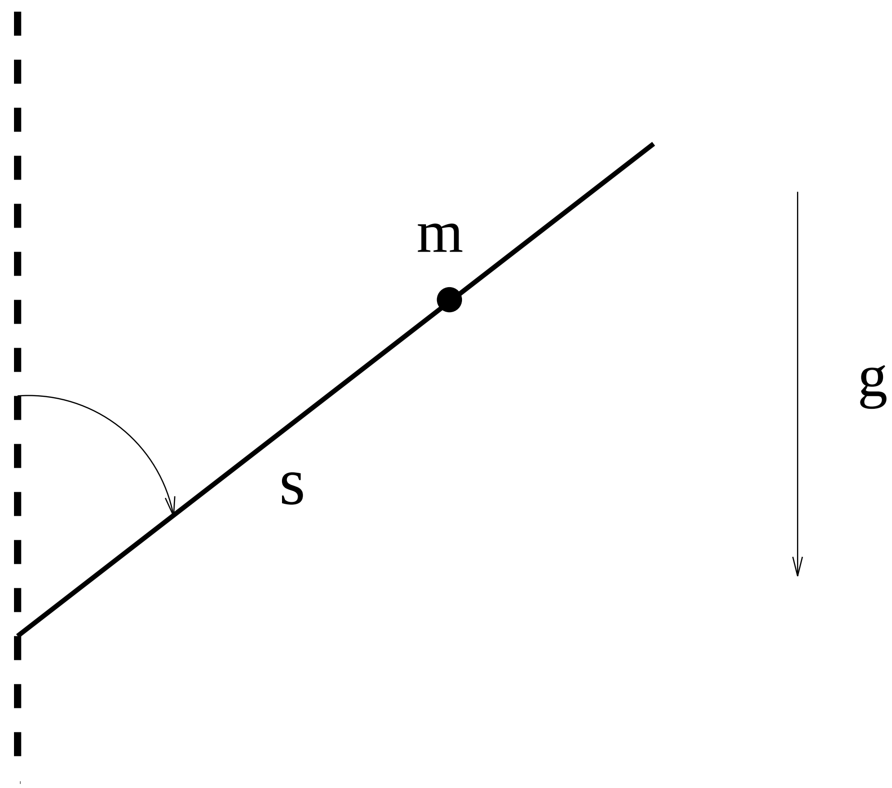
b. Determine the velocity of the vehicle as it passes the midpoint of the journey, C.

c. In an effort to speed up the journey, a propulsion system is attached to the vehicle that exerts a constant force of Mg on the vehicle, where M is the mass of the vehicle and g is the acceleration of gravity at the surface of the Earth. The force Mg is directed along A $\rightarrow$ B. If $\phi = \pi/2$, calculate the improved time of passage.

d. What is the velocity of the vehicle when it arrives at B?

Problem 3

A bead of mass m slides without friction along a straight wire that is rotating with constant angular velocity ω about a vertical axis. The wire makes a fixed angle $\theta_\circ$ with the rotation axis. Gravity acts downward.



a. Construct the Lagrangian for the bead using as generalized coordinate the distance s measured along the wire from the point of intersection with the rotation axis.

b. Obtain the equation of motion for s and use it to find the condition for an equilibrium circular orbit of radius $s_\circ \sin(\theta_\circ)$.

c. Use Lagrangian methods to find the force of constraint acting on the bead in the $\hat{\phi}$ direction. (ϕ is the azimuthal angle, with $\dot{\phi} = \omega$.)

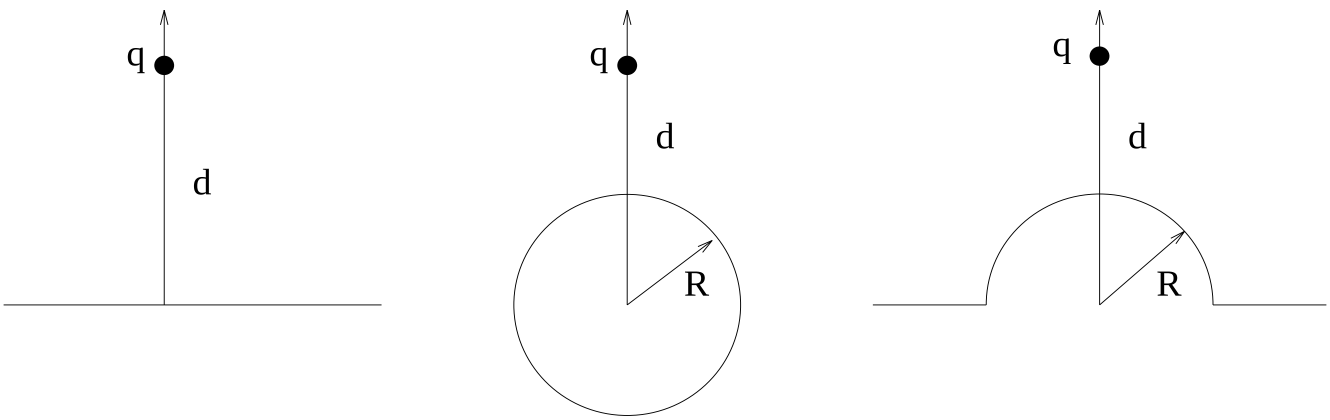
Problem 4

Calculate the force, **F**, felt by a point charge, **q**, when it is placed at a distance **d**, from

a. an infinite grounded conducting plane,

b. the center of a grounded conducting sphere of radius R.

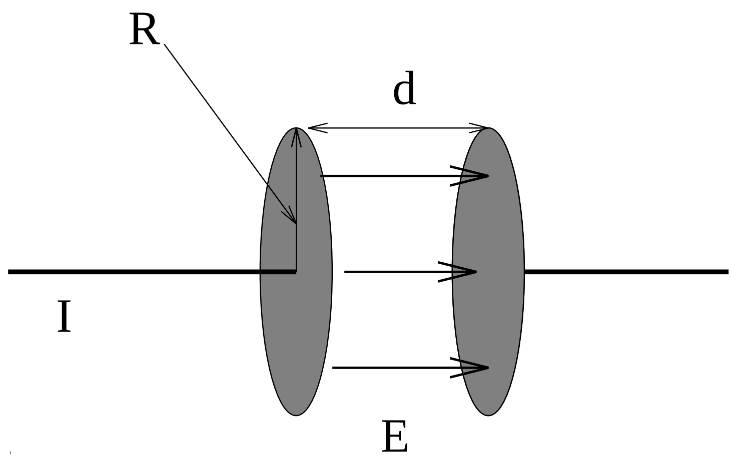
c. an infinite grounded conducting plane having a hemispherical bulge directly in front of q [i.e. in this case the distance between q and the closest point of the bulge is: (d-R)]. Also, evaluate the force in the limit when (d-R)$\rightarrow$ 0.



(a) (b) (c)

Problem 5

A parallel-plate capacitor composed of two parallel circular plates of radius R, separated by a distance d, is being charged in such a manner that the electric field between the plates $\mathbf{E}(t)$ is time dependent but spatially uniform between the plates. (Ignore the fringing of the lines of $\mathbf{E}$.)



a. Find an expression for the induced magnetic field $\mathbf{B}(r)$ at the radius $r < R$, in the region between the plates.

b. Calculate the total displacement current between the plates.

c. Calculate the Poynting vector $\mathbf{S}$ for $r = R$.

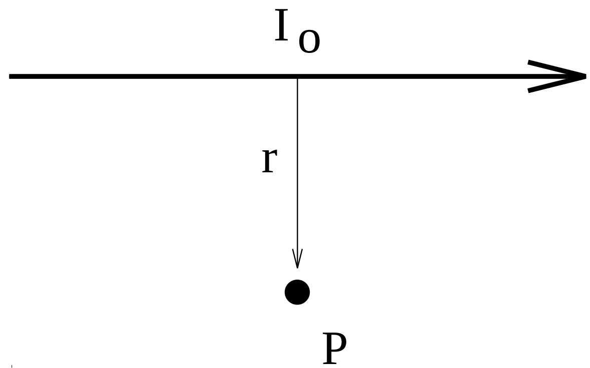
d. Sketch $\mathbf{B}$ and $\mathbf{S}$ for $r = R$.

e. The energy supplied by the power source in this circuit is being converted into what form?

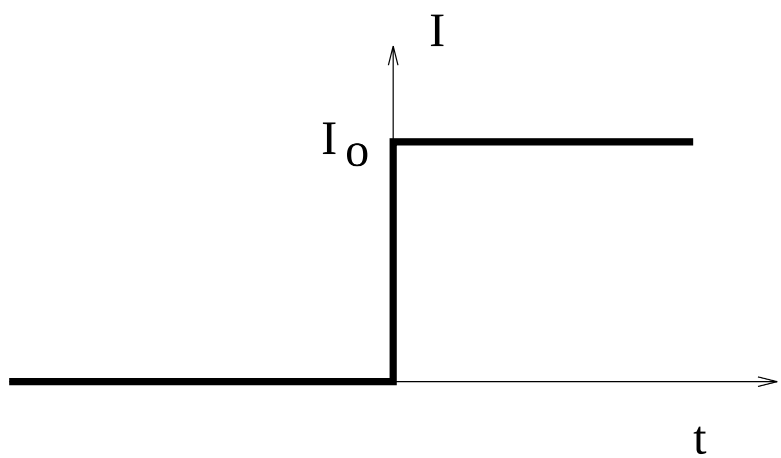
Problem 6

A point P is a distance r from a very long, straight wire. The wire is electrically neutral. Neglect any finite-length effects.

a. Assume that the wire carries a constant, steady current I_0. Determine the vector potential $\vec{A}(r)$ at point P, along with the fields $\vec{E}$ and $\vec{B}$ at P.



b. Now consider the situation where the current in the wire turns on abruptly at time $t = 0$, as shown in the figure below. (Assume that this change occurs at $t = 0$ for the entire length of wire.) Determine $\vec{A}(r, t)$ at P.



c. Use the result in (b) to determine $\vec{E}$ and $\vec{B}$ at P. Show that in the appropriate limit, these reduce to what you found in part (a).