Exam #: ______________________________

Printed Name: ______________________________

Signature: ______________________________

PHYSICS DEPARTMENT
UNIVERSITY OF OREGON
Masters Final Examination
and
Ph.D. Qualifying Examination, Part I
Monday, March 31, 2003, 1:00 p.m. to 5:00 p.m.

The examination pages are numbered in the upper left-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are twelve equally weighted questions, each beginning on a new page. Read all twelve questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic. **Calculators with stored equations or text are not allowed.** Dictionaries may be used if they have been approved by the proctor before the examination begins. **No other papers or books may be used.**

When you have finished the exam, come to the front of the room and hand your examination paper to the proctor; first put all problems in numerical order and staple them together.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand your exam paper in on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg (0.511 MeV/c^2)
Proton rest mass (m_p)	1.673×10^{-27} kg (938 MeV/c^2)
Neutron rest mass (m_n)	1.675×10^{-27} kg (940 MeV/c^2)
Planck's constant (h)	6.63×10^{-34} J-s
Speed of light in vacuum (c)	3.00×10^{8} m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Stefan-Boltzmann constant (σ)	5.67×10^{-8} J/(m^2-s-K^4)
Gravitational constant (G)	6.67×10^{-11} N-m^2/kg^2
Permeability of free space ($\mu_\circ$)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space ($\epsilon_\circ$)	8.85×10^{-12} F/m
Mass of Earth (M_E)	5.98×10^{24} kg
Equatorial radius of Earth (R_E)	6.38×10^{6} m
Radius of Sun (R_S)	6.96×10^{8} m
Mass of Sun (M_S)	1.99×10^{30} kg
Temperature of surface of the Sun (T_S)	$5,800$ K
Equatorial radius of Saturn (R_{saturn})	6.03×10^{7} m
Mass of Saturn (M_{saturn})	5.69×10^{26} kg
Temperature of surface of Saturn (T_{saturn})	123 K
Earth-Sun distance (R_{ES})	1.50×10^{11} m
Gravitational acceleration on Earth (g)	9.8 m/s^2
atomic mass unit	1.7×10^{-27} kg

Stirling's Formula

$$\ln(x!) = x\,\ln(x) - x - \ln\sqrt{2\pi x} + \mathcal{O}(1/x) \tag{1}$$

Larmor Radiation Formula

$$P = (\frac{2}{3c^3})q^2a^2 \tag{2}$$

Integrals

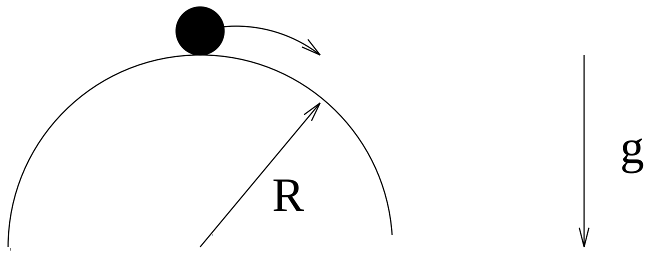
$$\int_{-\infty}^{\infty} dx\; x^{2n}\exp(-ax^2) = \frac{1 \times 3 \times 5 \times \cdots \times (2n-1)}{2^n a^n}\sqrt{\frac{\pi}{a}} \tag{3}$$

$$\int_0^{\infty} \frac{dx}{x}\; x^n\exp(-x) = \Gamma(n) \tag{4}$$

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln(x + \sqrt{x^2+a^2}) \tag{5}$$

Problem 1

A ball of mass m rests on top of a hemispherical mound of ice of radius R. A small gust of wind causes the ball to begin sliding without friction down the mound of ice.

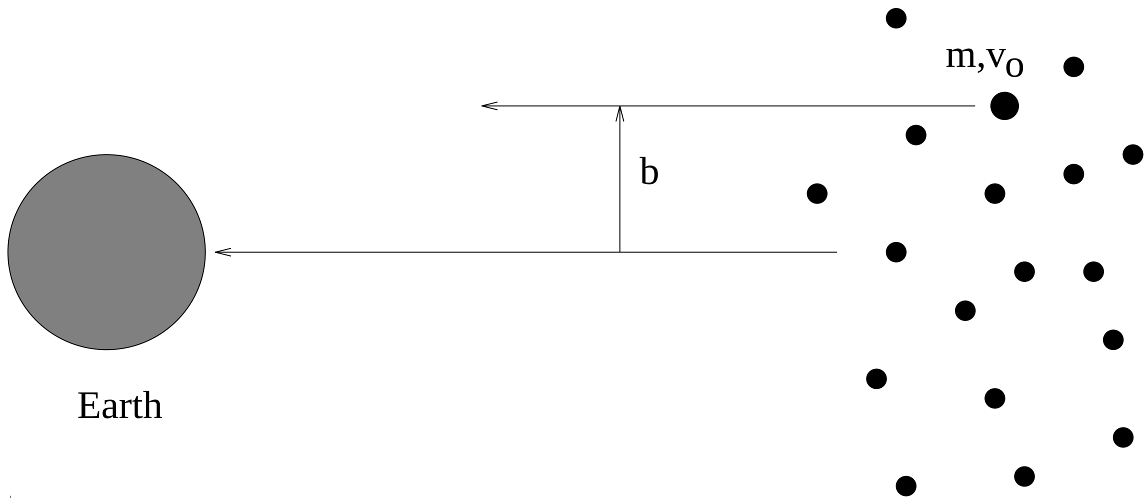


a. At what height from the ground does the ball leave the ice?

b. If there were friction between the ball and the ice, would this height be larger or smaller? The ball does not roll.

Problem 2

A swarm of meteors approaches the Earth from ∞. Each meteor has mass m and speed $v_\circ$.

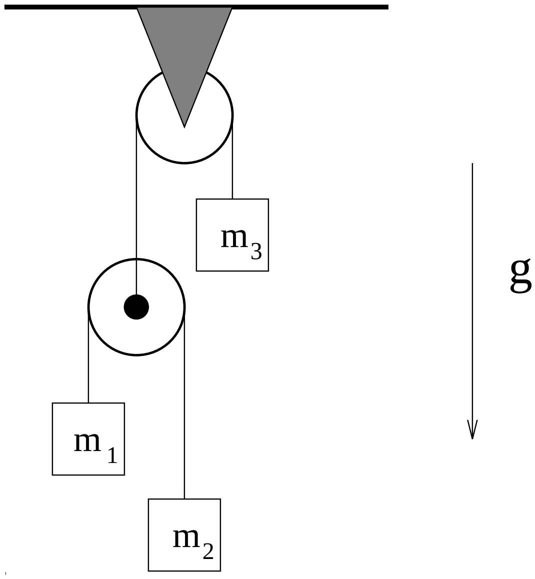


a. What is the angular momentum about the Earth for a meteor with impact parameter b?

b. For what range of impact parameter will an approaching meteor strike the Earth?

Ignore the influence of the Sun and other objects in the Solar System in your calculations.

Problem 3



Consider a pulley set-up with three masses as shown. The axle of the upper wheel is fixed, while the lower wheel is suspended from the upper string. Assume that there is no slack in the strings at any time.

a. Find the acceleration a_3 of mass m_3 in terms of g, m_1, m_2, and m_3.

b. Consider the special cases:

i. $m_3 \gg m_1, m_2$;

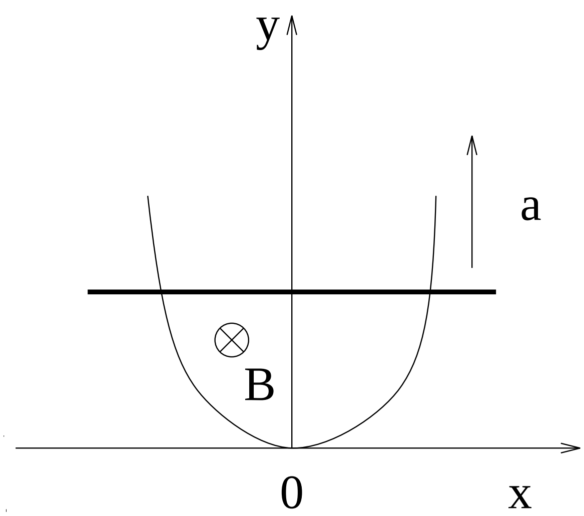
ii. $m_3 \to 0$;

iii. $m_1 = m_2 = m_3/2$.

Explain why your results are physically sensible for each case.

Problem 4

A wire is bent into a parabola $y = kx^2$, where k is a constant. The wire is located in a uniform magnetic field $\mathbf{B}$ which is perpendicular to the x-y plane. At $t = 0$ a metal conducting rod, which is parallel to the x axis, starts sliding upwards along the wire (i.e. increasing y) from the origin with constant acceleration a.



Derive an expression for the induced emf in the loop thus formed. Express your answers in terms of B, y, k and a.

Problem 5

The electric field in an electromagnetic wave is given by

$$\mathbf{E} = \mathbf{E}_0 \exp\left[i(\omega t - \mathbf{k}.\mathbf{r})\right],$$

and the magnetic field is described by a corresponding expression. This wave is incident on a free particle with mass m, and a charge q, and causes the particle to oscillate with an amplitude A. Assume that the speed of the particle is always $v \ll c$, and that $A \cdot \mathrm{k} \ll 1$.

a. What is the time averaged power radiated by the oscillating charge q, in terms of $\mathbf{E}$, ω, k, m, and q?

b. What is the total scattering cross section for this process?

Problem 6

An infinitely long straight wire connects the two poles of a battery, and carries a time independent current, I. The wire has a circular cross section of radius a, and has a conductivity σ.

a. Calculate the Poynting vector at the surface of the wire.

b. How much energy flows per unit length of the wire? What happens to this energy?

c. Answer in one simple sentence: What is the immediate source for the energy flow?

Problem 7

Assume that Saturn and the Sun can be regarded as sources of blackbody radiation. The radius of Saturn is R_{saturn}, and the mean radius of its orbit around the Sun is $d_{saturn} = 1.5 \times 10^{12}$m. From spectroscopic observations, we know that the surface temperature of Saturn is $T_{saturn} = 123$K. The energy per unit time impinging on Saturn from the Sun is not balanced by the power escaping from Saturn through blackbody radiation.

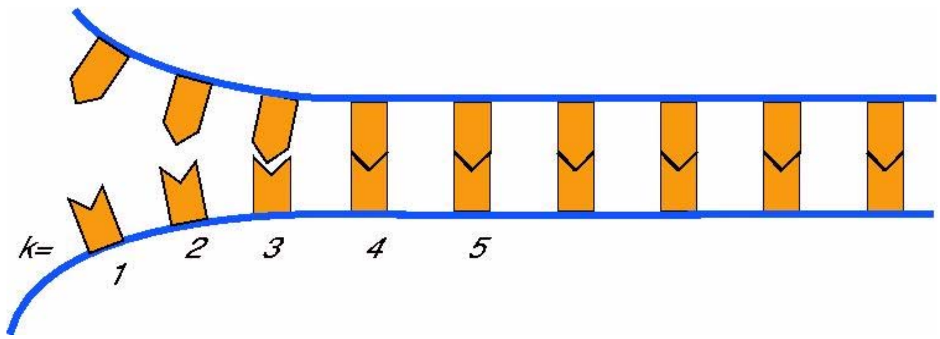
a. Write down a rate equation for the change of Saturn's total energy U per unit time, assuming that it loses energy only by blackbody radiation and it gains energy only through blackbody radiation from the Sun.

b. Compute the value of dU/dt in Watts.

c. As a possible cause of the nonzero dU/dt, we postulate slow gravitational contraction: modeling Saturn as a homogeneous sphere of radius R and mass M, its gravitational potential energy is $U_g = -\frac{3}{5}GM^2/R_{saturn}$. The (constant) mass of Saturn is $M = 5.69 \times 10^{26}$ kg, and the gravitational constant is $G = 6.67 \times 10^{-11}$, $\mathrm{N^2m^2kg^{-2}}$. If we assume $dU/dt = (1/2)dU_g/dt$, what is the rate dR_{saturn}/dt at which the radius of Saturn is decreasing? Why is only half of U_g available to be radiated?

Problem 8

A double-stranded DNA molecule is modeled as a ladder-like chain with N bonds between the two strands. Each bond, labeled by position index $k = 1, 2, \ldots N$, can be either open or closed. The energy of an open bond is $E = E_{open} = 1$ eV, while we set $E = 0$ for a closed bond. As in a zipper, the bonds can open only from the left end at $k = 1$, and a bond $k > 1$ can only open if its lefthand neighbor at position $k - 1$ is already open.



a. Show that the partition function Z for this model of the double helix is given by

$$Z(\beta, N) = \frac{1 - e^{-\beta(N+1)E_0}}{1 - e^{-\beta E_0}}$$

where $\beta = 1/(k_B T)$.

b. Write an expression for the expectation value of the number of open bonds, $\langle N_{open} \rangle$, at a given temperature T.

c. Show how can one calculate $\langle N_{open} \rangle$ as a derivative of $Z(\beta, N)$ from part (a)?

Problem 9

A rigid, closed and thermally isolated cylinder is divided into two unequal parts V_A and V_B by a frictionless, movable piston that conducts heat. The compartments contain ideal, monatomic gases A and B (without internal degrees of freedom). The total volume of the cylinder is $V_A + V_B = V$, and the number of particles in each volume are N_A and N_B. The gases have initial temperatures T_A and T_B.

a. Write the equation of state for an ideal, monatomic gas - i.e., the relation between pressure, volume, particle number and temperature

b. Write the total energy of the ideal gas in terms of particle number and temperature

c. If we let the two compartments come to equilibrium adiabatically by moving the piston until the forces on it are balanced, what are the final temperature $T_{A,f}$ and pressure $P_{A,f}$?

Problem 10

Consider an optics experiment in which a photon can be prepared in either state $|A\rangle$ or $|B\rangle$. The photon enters a device at time 0 and exits at time t in state

$$U(t,0)|A\rangle = \frac{1}{\sqrt{2}}(|\alpha\rangle + i|\beta\rangle)$$

or

$$U(t,0)|B\rangle = \frac{1}{\sqrt{2}}(|\beta\rangle + i|\alpha\rangle),$$

depending on the state in which it started.

a. For a photon initially in state A, what is the probability P_α for measuring it in the α state at time t? What is P_β for this photon? Calculate the same probabilities for a photon starting in the B state.

b. Now suppose we begin with two photons, one in the state A and the other in the state B. Assume they evolve independently of each other as they pass through the device. After time t, a measurement is performed on the photons. What are the probabilities for finding both in the α state, both in the β state, and one each in the α and β states?

Problem 11

Consider the Hamiltonian for a two electron system in a magnetic field

$$H = A\vec{S}_1 \cdot \vec{S}_2 + B(S_{1z} + S_{2z})$$

where $\vec{S}_{1,2} = \frac{\hbar}{2}\vec{\sigma}_{1,2}$ are the spin operators for electrons 1 and 2 respectively ($\vec{\sigma}$ are the Pauli matrices). Solve for the eigenvalues and eigenvectors of H. You may ignore the Pauli exclusion principle.

Problem 12

A particle of mass m is held in the ground state of a one dimensional harmonic oscillator potential centered about $x = 0$ such that the probability to find the particle at x in a small interval dx is given by

$$\mathcal{P}(x)dx = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-x^2}{2\sigma^2}} dx.$$

The potential is suddenly turned off at time $t = 0$.

a. Make a sketch showing what will happen to the probability density as a function of t.

b. Compute the probability to find the particle at x in an interval dx for $t > 0$ as a function of t, σ, $\hbar$ and the particle mass m.

Integral: if $Re(a) > 0$ and $Im(b) = 0$ then $\int_{-\infty}^{\infty} e^{-ay^2} e^{iby} dy = e^{-\frac{b^2}{4a}} \sqrt{\frac{\pi}{a}}$