Exam #: ______________________

Printed Name: ______________________

Signature: ______________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Ph.D. Qualifying Examination, PART III

Wednesday, April 3, 2002, 1:00 p.m. to 5:00 p.m.

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are six equally weighted questions, each beginning on a new page. Read all six questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic. **Calculators with stored equations or text are not allowed**. Dictionaries may be used if they have been approved by the proctor before the examination begins. **No other papers or books may be used.**

When you have finished, come to the front of the room and hand your examination paper to the proctor; first put all problems in numerical order and staple them together.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand your exam paper in on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	$1.60 \times 10^{-19}\,\mathrm{C}$
Electron rest mass (m_e)	$9.11 \times 10^{-31}\,\mathrm{kg}$ ($0.511\,\mathrm{MeV/c^2}$)
Proton rest mass (m_p)	$1.673 \times 10^{-27}\,\mathrm{kg}$ ($938\,\mathrm{MeV/c^2}$)
Neutron rest mass (m_n)	$1.675 \times 10^{-27}\,\mathrm{kg}$ ($940\,\mathrm{MeV/c^2}$)
W^+ rest mass (m_W)	$80.4\,\mathrm{GeV/c^2}$
Planck's constant (h)	$6.63 \times 10^{-34}\,\mathrm{J\cdot s}$
Speed of light in vacuum (c)	$3.00 \times 10^{8}\,\mathrm{m/s}$
Boltzmann's constant (k_B)	$1.38 \times 10^{-23}\,\mathrm{J/K}$
Gravitational constant (G)	$6.67 \times 10^{-11}\mathrm{N\cdot m^2/kg^2}$
Permeability of free space (μ_0)	$4\pi \times 10^{-7}\,\mathrm{H/m}$
Permittivity of free space (ϵ_0)	$8.85 \times 10^{-12}\,\mathrm{F/m}$
Mass of earth (M_E)	$5.98 \times 10^{24}\,\mathrm{kg}$
Equatorial radius of earth (R_E)	$6.38 \times 10^{6}\,\mathrm{m}$
Radius of sun (R_S)	$6.96 \times 10^{8}\,\mathrm{m}$
Temperature of surface of sun (T_S)	$5.8 \times 10^{3}\,\mathrm{K}$
Earth - sun distance (R_{ES})	$1.50 \times 10^{11}\,\mathrm{m}$
Density of iron at low temperature (ρ_{Fe})	$7.88 \times 10^{3}\,\mathrm{kg/m^3}$
Classical electron radius (r_0)	$2.82 \times 10^{-15}\,\mathrm{m}$
Gravitational acceleration on Earth (g)	$9.8\,\mathrm{m/s^2}$
Atomic mass unit	$1.7 \times 10^{-27}\,\mathrm{kg}$
Specific heat of oxygen (c_V)	$21.1\,\mathrm{J/mole\cdot K}$
Specific heat of oxygen (c_P)	$29.4\,\mathrm{J/mole\cdot K}$

Stirling's Formula

$$\ln(x!) = x\ln(x) - x - \ln\left(\sqrt{2\pi x}\right) + \mathcal{O}(1/x)$$

Integrals

$$\begin{aligned} \int_{-\infty}^{\infty} dx\, x^{2n}\, e^{-ax^2} &= \frac{1 \times 3 \times 5 \times \cdots \times (2n-1)}{2^n\, a^n}\sqrt{\frac{\pi}{a}} \\ \int_0^{\infty} \frac{dx}{x}\, x^n\, e^{-x} &= \Gamma(n) \end{aligned}$$

Problem 1

A point mass m moving vertically under the gravitational acceleration g follows the one-dimensional Schrödinger equation with a potential $V(x) = -mgx$, where x is the height of the point mass.

a) Show by direct substitution that the time-dependent Schrödinger equation is solved by a wave packet of the form

$$\psi(s,t) = \int_{-\infty}^{+\infty} dk\; e^{ikx - i\hbar^2 k^3/(6m^2 g)}\; A\left(\frac{\hbar k}{mg} - t\right),$$

where $A(z)$ is a (generally complex-valued) function of one real variable. The form of $A(z)$ is arbitrary. We require only that it should be continuously differentiable and decay to zero as $z \to \pm\infty$. No integrals have to be calculated explicitly in this proof.

b) Write down an integral to determine the expectation value of the momentum p from the function A.

c) Now choose the particular form

$$A\left(\frac{\hbar k}{mg} - t\right) = \exp\left\{-b^2\left(\frac{\hbar k}{mg} - t + t_0\right)^2 + i\frac{mg^2}{6\hbar}\left(\frac{\hbar k}{mg} - t\right)^3\right\},$$

where b and t_0 are real constants. Show that the expectation value of the momentum, $\langle p\rangle$, follows Newton's second law for the time dependence of the momentum of a mass m under acceleration $-g$.

Problem 2

This problem is related to Van der Waals interactions. Consider two hydrogen atoms in their ground states. If one assumes that the nuclei of the two atoms are a fixed distance r apart (where r is much greater than the Bohr radius), then the dipole-dipole interaction between the two atoms is given by

$$V(r) = \left\{ \vec{d}_1 \cdot \vec{d}_2 - 3(\vec{d}_1 \cdot \hat{n})(\vec{d}_2 \cdot \hat{n}) \right\} / r^3,$$

where $\vec{d}_1$ and $\vec{d}_2$ are the atomic dipole moments of the two atoms and $\hat{n}$ is a unit vector in the direction from atom 1 to atom 2.

a) Show that the first order correction to the energy of the system due to the dipole-dipole interaction is zero.

b) Write down the second order correction to the energy of the system due to the dipole-dipole interaction, assuming that one knows the energy structure and dipole matrix elements of the atom. Is this interaction attractive or repulsive? Explain.

Problem 3

Consider a simple model of the atomic nucleus as a cubical box with zero potential inside the box and infinite potential outside. The nucleons are thus confined to the box. In the model, they do not interact with each other: the strong interactions are modelled as producing the box potential. The length L of the side of the cube can be taken to be the diameter of the nucleus. Investigate the iron-56 nucleus with 28 protons and 28 neutrons. Assume a nuclear diameter of $L = 10^{-14}$ m. Assume that the nucleus is in its ground state.

a) Tabulate the single particle states of the nucleons in terms of the quantum numbers $\{n_x, n_y, n_z\}$, beginning with $\{1, 1, 1\}$ in order of energy, computing the value of $n_x^2 + n_y^2 + n_z^2$ for each single particle state.

b) Calculate the total kinetic energy of the nucleus in its ground state.

c) The electric interactions of the nucleons have so far not been included in the model. Make a simple estimate of the total electrostatic energy of the nucleus.

Problem 4

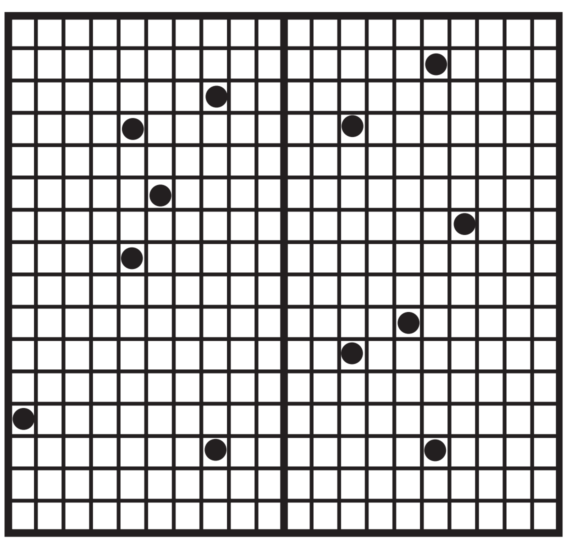
Consider the surface of a crystalline solid upon which indistinguishable atoms are bound to particular atomic surface sites. These atoms can move fairly freely from site to site; however, no more than a single atom can occupy a given site. Imagine that the surface is divided into two regions, each with N sites, by a barrier that is impenetrable to the atomic motion. Suppose that initially each of the two regions contains the same number n of bound surface atoms.

a) With the partition in place, find the number of distinct (micro-) states for the total system.

b) Now the partition is removed, thus allowing atoms to cross over between the two regions. Find the number of states for the combined system.

c) In the case that N, n, and $N - n$ are all very large, find the entropy for the combined system before and after removing the partition.

d) Discuss *qualitatively* whether or not the entropy would remain approximately the same after removing the barrier if, initially, there were twice as many atoms on one side of the barrier as the other.



Region I Region II

Problem 5

Consider an equilibrium process where electrons and positrons are generated thermally out of a photon gas at temperature T. (Recall that two photons can turn into an electron-positron pair.) Assume that the densities are equal ($n_+ = n_- \equiv n$), but are low enough that the electrons and the positrons can each be treated as a classical ideal gas. It may be helpful to recall the expression for the partition function, Z, of the classical ideal gas:

$$\ln(Z) = N\left[\ln(V/N) + \frac{3}{2}\ln(k_B T) + \frac{3}{2}\ln\left(2\pi m/h^2\right) + 1\right].$$

a) Find an expression for n as a function of temperature.

b) At what characteristic temperature would we observe a significant spontaneous generation of electrons and positrons? (Please provide your answer in Kelvins).

Problem 6

These questions concern heat transfers.

a) When an amount ΔQ of heat is transferred to a body at temperature T there is a consequent change ΔS in the entropy of the body. What is the relation between ΔS, ΔQ, and T?

b) A block of material with a temperature independent heat capacity $C = 500$ J/K is initially at a temperature $T_0 = 600$ K. It is then placed in a lake at a temperature $T_L = 300$ K and allowed to come into thermal equilibrium with the lake. Find the change in entropy of the block, the lake, and the total system.

c) An identical block at an initial at temperature $T_0 = 600$ K is placed in a large geothermally heated pool of water that is at the boiling point of water, $T_B = 100^o$ C. The block is allowed to come into thermal equilibrium with the pool. It is then placed in the lake at a temperature $T_L = 300$ K and allowed to come into thermal equilibrium with the lake. Find the change in entropy of the block, the pool, the lake, and the total system.

d) Explain how the entropy change involved in cooling the block from 600 K to 300 K could be reduced to nearly zero.