Exam #: ________________________

Printed Name: ________________________

Signature: ________________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Ph.D. Qualifying Examination, PART II

Tuesday, April 2, 2002, 1:00 p.m. to 5:00 p.m.

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are six equally weighted questions, each beginning on a new page. Read all six questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic. **Calculators with stored equations or text are not allowed**. Dictionaries may be used if they have been approved by the proctor before the examination begins. **No other papers or books may be used.**

When you have finished, come to the front of the room and hand your examination paper to the proctor; first put all problems in numerical order and staple them together.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand your exam paper in on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	$1.60 \times 10^{-19}\,\mathrm{C}$
Electron rest mass (m_e)	$9.11 \times 10^{-31}\,\mathrm{kg}$ ($0.511\,\mathrm{MeV/c^2}$)
Proton rest mass (m_p)	$1.673 \times 10^{-27}\,\mathrm{kg}$ ($938\,\mathrm{MeV/c^2}$)
Neutron rest mass (m_n)	$1.675 \times 10^{-27}\,\mathrm{kg}$ ($940\,\mathrm{MeV/c^2}$)
W^+ rest mass (m_W)	$80.4\,\mathrm{GeV/c^2}$
Planck's constant (h)	$6.63 \times 10^{-34}\,\mathrm{J \cdot s}$
Speed of light in vacuum (c)	$3.00 \times 10^{8}\,\mathrm{m/s}$
Boltzmann's constant (k_B)	$1.38 \times 10^{-23}\,\mathrm{J/K}$
Gravitational constant (G)	$6.67 \times 10^{-11}\mathrm{N \cdot m^2/kg^2}$
Permeability of free space (μ_0)	$4\pi \times 10^{-7}\,\mathrm{H/m}$
Permittivity of free space (ϵ_0)	$8.85 \times 10^{-12}\,\mathrm{F/m}$
Mass of earth (M_E)	$5.98 \times 10^{24}\,\mathrm{kg}$
Equatorial radius of earth (R_E)	$6.38 \times 10^{6}\,\mathrm{m}$
Radius of sun (R_S)	$6.96 \times 10^{8}\,\mathrm{m}$
Temperature of surface of sun (T_S)	$5.8 \times 10^{3}\,\mathrm{K}$
Earth - sun distance (R_{ES})	$1.50 \times 10^{11}\,\mathrm{m}$
Density of iron at low temperature (ρ_{Fe})	$7.88 \times 10^{3}\,\mathrm{kg/m^3}$
Classical electron radius (r_0)	$2.82 \times 10^{-15}\,\mathrm{m}$
Gravitational acceleration on Earth (g)	$9.8\,\mathrm{m/s^2}$
Atomic mass unit	$1.7 \times 10^{-27}\,\mathrm{kg}$
Specific heat of oxygen (c_V)	$21.1\,\mathrm{J/mole \cdot K}$
Specific heat of oxygen (c_P)	$29.4\,\mathrm{J/mole \cdot K}$

Stirling's Formula

$$\ln(x!) = x\ln(x) - x - \ln\left(\sqrt{2\pi x}\right) + \mathcal{O}(1/x)$$

Integrals

$$\int_{-\infty}^{\infty} dx\, x^{2n}\, e^{-ax^2} = \frac{1 \times 3 \times 5 \times \cdots \times (2n-1)}{2^n\, a^n} \sqrt{\frac{\pi}{a}}$$

$$\int_0^{\infty} \frac{dx}{x}\, x^n\, e^{-x} = \Gamma(n)$$

Problem 1

Consider a mechanical system described by a single coordinate $x(t)$ that evolves according to the Lagrangian

$$L = \frac{1}{2} m \frac{\dot{x}^2}{1 - (x/a)^2}.$$

a) Find the Euler-Lagrange equation of motion for $x(t)$.

b) The system begins at $t = 0$ with initial conditions $x(0) = a/2$, $\dot{x}(0) = 0$. What will be its subsequent behavior?

c) [*This part slightly revised from the original. Ed.*] Now suppose that the system begins at $t = 0$ with initial conditions $x(0) = 0$, $\dot{x}(0) = v_0$. What will be its behavior at later times? [Hints: are there any conserved quantities? If you get an integral that you doesn't look familiar, try a trigonometric substitution.]

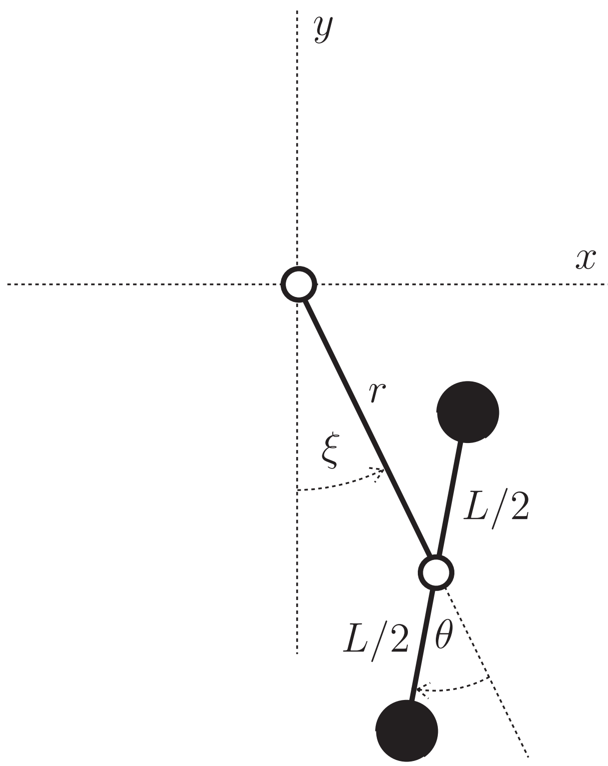
Problem 2

Two particles, each having the same mass m, move in the $\{x, y\}$ plane, where y is the vertical direction. The particles are joined by a rigid, massless rod of length L. The center of the rod is connected by a frictionless pivot to the end of a second rigid, massless rod, which has a length r. The top end of the second rod is connected to a fixed support structure by a frictionless pivot. The system can be characterized by the two angles ξ and θ shown in the figure.

a) Write down the Lagrangian for this system and use it to write the Euler-Lagrange equations that determine the time evolution of $\xi(t)$ and $\theta(t)$.

b) Find the general solution of these equations for which $\xi(t) \equiv 0$.

c) Qualitatively discuss the nature of the motion for which $\xi(t)$ is not identically equal to zero.



Problem 3

The earth is continually bombarded by matter – dust and meteors – from space. This bombardment has a small effect on the earth's rotation.

a) Show that the moment of inertia of a thin spherical shell of mass m and radius R is $(2/3)\, mR^2$.

b) Show that the moment of inertia of a solid sphere of mass M and radius R is $(2/5)\, MR^2$.

c) Suppose that in a given year a mass m of material falls on earth randomly from all directions. Calculate the fractional change in the length of the day per year in terms of m and the mass M of the earth.

Problem 4

Consider a cylindrically symmetric current density given by

$$\begin{aligned} j_x(\vec{x}) &= 0 \\ j_y(\vec{x}) &= 0 \\ j_z(\vec{x}) &= j(r), \end{aligned}$$

where $r = \sqrt{x^2 + y^2}$.

a) From Ampère's law, find the magnetic field $\vec{B}(\vec{x})$ in terms of an integral over $j(r)$.

b) Determine and sketch $\vec{B}(\vec{x})$ for the special condition

$$j(r) = \begin{cases} I_0/(\pi r_0^2) & \text{for } r \leq r_0 \\ 0 & \text{for } r > r_0 \end{cases},$$

where I_0 and r_0 are constants.

c) For the current $\vec{j}$ specified in part b), show that a vector potential $\vec{A}(\vec{x})$ exists such that

$$\vec{B}(\vec{x}) = \nabla \times \vec{A}(\vec{x})$$

with the condition that

$$\vec{A}(\vec{x}) = 0 \qquad \text{for} \quad r = r_0.$$

Determine and sketch $\vec{A}(\vec{x})$ explicitly.

Problem 5

A particle of charge q and mass m is released at rest from the origin in the presence of a uniform electric field $\boldsymbol{E} = E_0\hat{k}$ and a uniform magnetic field $\boldsymbol{B} = B_0\hat{\jmath}$, where $\{\hat{\imath}, \hat{\jmath}, \hat{k}\}$ are unit vectors along the x, y, and z axes respectively.

a) Qualitatively explain the motion of the particle after it has been released, and sketch the expected trajectory.

b) An elegant relativistic solution can be obtained by performing a Lorentz transformation from the original frame S into a frame S' where the electric field vanishes. Find the boost $\boldsymbol{\beta}$ needed to make $\boldsymbol{E'} = 0$. (Assume $E_0 < B_0$.)

Hint: If a four-vector transforms as $\boldsymbol{p'} = \boldsymbol{\Lambda p}$, then the field tensor transforms as $\boldsymbol{F'} = \boldsymbol{\Lambda F \Lambda^T}$, where

$$\boldsymbol{F} = \left\{ \begin{array}{cccc} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & B_z & -B_y \\ -E_y/c & -B_z & 0 & B_x \\ -E_z/c & B_y & -B_x & 0 \end{array} \right\},$$

and $\boldsymbol{\Lambda(\beta)}$ is the Lorentz transformation matrix. (If you wish, you may work in units where $c = 1$.)

c) Solve for the trajectory of the particle in the frame S'. Do **not** assume non-relativistic motion, although you may assume that the particle does not radiate (*ie.*, ignore radiation damping). Explain how this solution compares to what you described in part a).

Problem 6

Two mutually coherent, monochromatic, plane light waves with equal electric field amplitudes, but *unequal* angular frequencies ω_1 and ω_2, co-propagate in free space in the z-direction. Assume $|\omega_2 - \omega_1|$ is small compared to ω_1 or ω_2. Wave '1' is linearly polarized in the x-direction and wave '2' is right-circularly polarized. (Mutually coherent waves can be formed, for example, by splitting a beam into two by use of a half-silvered mirror and frequency shifting one of them by means of the Doppler effect.)

a) Describe quantitatively the intensity of the light as a function of time striking a fast detector placed with its surface perpendicular to the z-axis. (The detector's response time is less than $|\omega_2 - \omega_1|^{-1}$.)

b) Describe qualitatively the behavior of the intensity if one field is monochromatic but the other has a spectral width that equals roughly three-fourths of the frequency difference $|\omega_2 - \omega_1|$. (This could result from one field traveling through a fluctuating medium.)