Exam #: ______________________

Printed Name: ______________________

Signature: ______________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Ph.D. Qualifying Examination, PART III

Quantum Mechanics and Statistical Mechanics

Friday, April 6, 2000, 1:00 p.m. to 5:00 p.m.

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are six equally weighted questions, each beginning on a new page. Read all six questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic. **Calculators with stored equations or text are not allowed**. Dictonaries may be used if they have been approved by the proctor before the examination begins. **No other papers or books may be used.**

When you have finished, come to the front of the room and hand your examination paper to the proctor; first put all problems in numerical order and staple them together.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand your exam paper in on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg (0.511 MeV/c^2)
Proton rest mass (m_p)	1.673×10^{-27} kg (938 MeV/c^2)
Neutron rest mass (m_n)	1.675×10^{-27} kg (940 MeV/c^2)
W^+ rest mass (m_W)	80.4 GeV/c^2
Planck's constant (h)	6.63×10^{-34} J$\cdot$ s
Speed of light in vacuum (c)	3.00×10^{8} m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Gravitational constant (G)	6.67×10^{-11}N$\cdot$ m^2/kg^2
Permeability of free space (μ_0)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space (ϵ_0)	8.85×10^{-12} F/m
Mass of Earth (M_E)	5.98×10^{24} kg
Equatorial radius of Earth (R_E)	6.38×10^{6} m
Avagadro's number (N_A)	6.02×10^{29} m^{-3}
Density of Fe at low temperature (ρ_{Fe})	7.88×10^{3} kg/m^3
Classical electron radius (r_0)	2.82×10^{-15} m
Gravitational acceleration on Earth (g)	9.8 m/s^2
Atomic mass unit	1.7×10^{-27} kg
Specific heat of oxygen (c_V)	21.1 J/mole$\cdot$ K
Specific heat of oxygen (c_P)	29.4 J/mole$\cdot$ K

Some information

$$\vec{\nabla} \cdot \vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 A_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\, A_\theta\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi} A_\phi$$

$$Y_{lm} = \left[\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}\right]^{1/2} P_l^m(\cos\theta)\, e^{im\phi}$$

$$\begin{aligned} P_0(\cos\theta) &= 1 \\ P_1(\cos\theta) &= \cos\theta \\ P_2(\cos\theta) &= \tfrac{1}{2}(3\cos^2\theta - 1) \\ P_3(\cos\theta) &= \tfrac{1}{2}(5\cos^3\theta - 3\cos\theta) \\ P_4(\cos\theta) &= \tfrac{1}{8}(35\cos^4\theta - 30\cos^2\theta + 3) \end{aligned}$$

$$P_l^m(x) = (-1)^m (1-x^2)^{m/2} \frac{d^m}{dx^m} P_l(x)$$

$$\left[\frac{d^2}{dx^2} + \frac{1}{x}\frac{d}{dx} + \left(1 - \frac{\nu^2}{x^2}\right)\right] J_\nu(x) = 0$$

Problem 1

Suppose an electron is described by the wavefunction

$$R(r)\left[\sqrt{\frac{3}{4}}Y_2^2(\theta,\phi)\chi_- + \frac{1}{2}Y_2^{-1}(\theta,\phi)\chi_+\right],$$

where $R(r)$ is a normalized radial function, $Y_2^2(\theta,\phi)$ and $Y_2^{-1}(\theta,\phi)$ are spherical harmonics, and χ_+ and χ_- are spinors for up and down electron spin, respectively.

a. Write down an expression for the probability of finding this particle in a spin up state at a distance from the origin lying between r and $r+dr$.
b. Write down an expression for the probability of finding this particle at a distance from the origin lying between r and $r+dr$ *irrespective* of its spin state.
c. If you measure the z-component of total angular momentum, $L_z + S_z$, for this state, what values might you get and what is the probability for each?

Problem 2

An infinitely deep potential well,

$$V(x) = \begin{cases} \infty, & x < 0, \\ 0, & 0 \leq x \leq a, \\ \infty, & x > a, \end{cases}$$

contains a particle of mass m.

a. Starting from the time independent Schrödinger equation, determine the normalized wavefunction of the general energy eigenstate.

b. What are the energy levels of this system?

c. Suppose a particle was in the ground state when the width of the potential well *suddenly* expanded from a to $2a$. What is the probability that the particle remains in the ground state?

Problem 3

Consider a two-state atom with energy level difference $\hbar\omega_0$, driven through the electric dipole interaction (dipole moment $\mu = \langle 1|ex|2\rangle = \langle 2|ex|1\rangle$) by a near-resonant classical electromagnetic field $E\cos(\omega t)$.

a. Write down the Hamiltonian of the system in a 2×2 matrix representation.

b. Assume the field is turned on at $t = 0$, at which time the atom is in the lower state. Use time-dependent perturbation theory to find the probability for the atom to be in the upper state after a short time interval τ. (You will find the contribution from nonresonant terms is much smaller than that from resonant terms.)

c. For near-resonant excitation, perturbation theory becomes inadequate for sufficiently long times. Ignore the nonresonant terms and hence determine the upper state population as a function of time when the field is exactly resonant with the atomic transition.

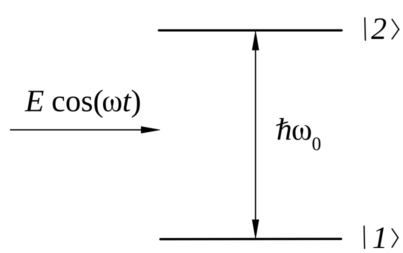

Figure 1: **Figure for Problem 3**

Problem 4

Estimate the bulk modulus of solid Fe, atomic mass = 56, at low temperature. Base your estimate on the assumption that the order of magnitude of the bulk modulus is given by the pressure of a "degenerate electron gas" containing all conduction electrons in the solid; i.e., as a crude approximation, assume that the negative pressure from the Fe lattice is independent of volume. Assume that each Fe atom contributes one conduction electron. Express your answer in units of N/m^3.

Recall: the bulk modulus, K, is defined by

$$\frac{1}{K} \equiv -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_{T,N},$$

where P and V are the applied pressure and sample volume, respectively.

Problem 5

Electron-positron pairs are produced thermally in the vacuum at sufficiently high temperatures. Because of charge conjugation symmetry the chemical potential $\mu = 0$.

a. Derive an expression for the probability that a state of energy ϵ is occupied by a fermion at temperature T.

b. Derive an integral for the equlibrium density of electrons at temperature T in terms of the rest mass of the electron, neglecting interactions between the particles.

c. Approximate the integral in the low temperature limit to obtain the leading temperature dependence of the electron density.

Problem 6

Consider a dilute suspension of particles with magnetic moment μ at temperature T. This suspension is placed in a non-uniform magnetic field whose magnitude, $B(z)$, depends only on z. Treat the magnetic moment classically and ignore the effect of gravity.

a. Find an expression for the ratio of the density of particles at two different positions, $n(z_2)/n(z_1)$.

b. If $B(z_2) > B(z_1)$, is $n(z_2)$ greater than or less than $n(z_1)$? Give a physical reason for your result.