Exam #: ______________________

Printed Name: ______________________

Signature: ______________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Ph.D. Qualifying Examination, PART II

Mechanics and Electromagnetism

Thursday, April 5, 2001, 1:00 p.m. to 5:00 p.m.

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are six equally weighted questions, each beginning on a new page. Read all six questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic. **Calculators with stored equations or text are not allowed**. Dictonaries may be used if they have been approved by the proctor before the examination begins. **No other papers or books may be used.**

When you have finished, come to the front of the room and hand your examination paper to the proctor; first put all problems in numerical order and staple them together.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand your exam paper in on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg ($0.511\,\mathrm{MeV/c^2}$)
Proton rest mass (m_p)	1.673×10^{-27} kg ($938\,\mathrm{MeV/c^2}$)
Neutron rest mass (m_n)	1.675×10^{-27} kg ($940\,\mathrm{MeV/c^2}$)
W^+ rest mass (m_W)	$80.4\,\mathrm{GeV/c^2}$
Planck's constant (h)	6.63×10^{-34} J$\cdot$ s
Speed of light in vacuum (c)	3.00×10^{8} m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Gravitational constant (G)	$6.67 \times 10^{-11}\mathrm{N\cdot m^2/kg^2}$
Permeability of free space (μ_0)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space (ϵ_0)	8.85×10^{-12} F/m
Mass of Earth (M_E)	5.98×10^{24} kg
Equatorial radius of Earth (R_E)	6.38×10^{6} m
Avagadro's number (N_A)	$6.02 \times 10^{29}\,\mathrm{m^{-3}}$
Density of Fe at low temperature (ρ_{Fe})	$7.88 \times 10^{3}\,\mathrm{kg/m^3}$
Classical electron radius (r_0)	2.82×10^{-15} m
Gravitational acceleration on Earth (g)	$9.8\,\mathrm{m/s^2}$
Atomic mass unit	1.7×10^{-27} kg
Specific heat of oxygen (c_V)	21.1 J/mole$\cdot$ K
Specific heat of oxygen (c_P)	29.4 J/mole$\cdot$ K

Some information

$$\vec{\nabla} \cdot \vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi} A_\phi$$

$$Y_{lm} = \left[\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}\right]^{1/2} P_l^m(\cos\theta)\, e^{im\phi}$$

$$\begin{aligned} P_0(\cos\theta) &= 1 \\ P_1(\cos\theta) &= \cos\theta \\ P_2(\cos\theta) &= \tfrac{1}{2}(3\cos^2\theta - 1) \\ P_3(\cos\theta) &= \tfrac{1}{2}(5\cos^3\theta - 3\cos\theta) \\ P_4(\cos\theta) &= \tfrac{1}{8}(35\cos^4\theta - 30\cos^2\theta + 3) \end{aligned}$$

$$P_l^m(x) = (-1)^m (1-x^2)^{m/2}\frac{d^m}{dx^m} P_l(x)$$

$$\left[\frac{d^2}{dx^2} + \frac{1}{x}\frac{d}{dx} + \left(1 - \frac{\nu^2}{x^2}\right)\right] J_\nu(x) = 0$$

Problem 1

A linear symmetric molecule consists of three atoms on one straight line as shown in the figure. The forces between the molecules are modeled by two identical springs of spring constant k. Consider only vibrations along the line of the molecules.

a. Write down the Lagrangian that describes the molecule.
b. Find the characteristic frequencies of the moelcule's oscillation.
c. Describe briefly the motion associated with each frequency.

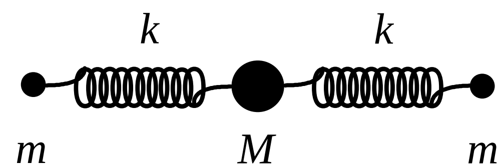

Figure 1: **Figure for Problem 1**

Problem 2

A particle of mass m moves on a circular orbit of radius $r = 1/\alpha$ in the central force field given by the potential

$$\Phi(r) = -k\frac{\exp(-\alpha r)}{r};$$

k and α are constants.

a. Calculate and plot the effective potential for radial motion.
b. Show that the orbit at $r = 1/\alpha$ is stable to radial perturbations.
c. Considering other values of r, for what range of r's are circular orbits stable?

Problem 3

A point mass m can slide without friction on a circular wire loop of radius a, as shown in the figure below. The loop rotates about the z-axis with angular velocity Ω, and gravity acts in the negative z-direction.

a. Determine the system's Lagrangian, adopting the angle θ as the generalized coordinate.
b. Determine the equilibrium positions.
c. Discuss the stability of the equilibrium positions; determine an Ω_0 such that there is a change of stability at $\Omega = \Omega_0$.
d. For $\Omega > \Omega_0$, determine the frequency of the small oscillations about the stable equilibrium position.

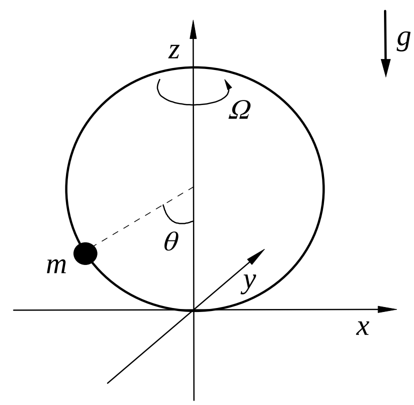

Figure 2: **Figure for Problem 3**

Problem 4

A particle with charge q moves along the z axis in the $+z$ direction with a constant speed v that is a substantial fraction of the speed of light. It crosses $z = 0$ at time $t = 0$. At time $t = 0$, derive an expression for the electric field $\boldsymbol{E}(\boldsymbol{x})$ produced by this particle.

HINT: You may choose units with $c = 1$ if you so wish.

Problem 5

A dielectric medium can be viewed as a collection of classical damped harmonic oscillators. Assume that for each oscillator an electron is connected to a fixed ion by a harmonic force of frequency ω_0 and damping constant γ_0. There are n such oscillators per unit volume.

a. Write down the equation of motion for the electron when driven by a monochromatic electric field of frequency ω.

b. Determine the electric polarization and the dielectric constant $\epsilon(\omega)/\epsilon_0$.

c. Free electrons in a metal can be considered as a limiting case of this model. Consider the appropriate limiting case and hence determine expressions for the frequency ranges in which metals become (i) transparent to electromagnetic waves, (ii) good reflectors of electromagnetic waves. (You may ignore damping.)

Problem 6

As shown in the figure, a charge $+q$ is placed at each of the four locations $\boldsymbol{x}_1 = (1, 0, 0)$, $\boldsymbol{x}_2 = (-1, 0, 0)$, $\boldsymbol{x}_3 = (0, 1, 0)$, $\boldsymbol{x}_4 = (0, -1, 0)$, and a charge $-2q$ at each of the additional locations $\boldsymbol{x}_5 = (0, 0, 1)$, $\boldsymbol{x}_6 = (0, 0, -1)$. Find the asymptotic form, up to a multiplicative constant, of the electric field $\boldsymbol{E}(\boldsymbol{x})$—i.e., for $|\boldsymbol{x}| \gg 1$, $\boldsymbol{E}(\boldsymbol{x})$ has a form

$$\boldsymbol{E}(\boldsymbol{x}) \sim C\,\boldsymbol{E}_0(\boldsymbol{x})[1 + \mathcal{O}(1/|\boldsymbol{x}|)].$$

You are asked to find $\boldsymbol{E}_0(\boldsymbol{x})$, but not C.

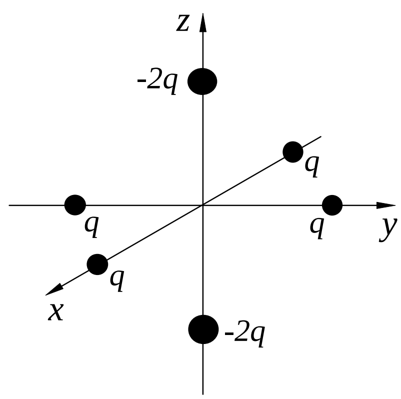

Figure 3: **Figure for Problem 6**