Exam #: ____________________

Printed Name: ____________________

Signature: ____________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Master's Final Examination, Ph.D. Qualifying Examination, PART I

Wednesday, April 4, 2001, 1:00 p.m. to 5:00 p.m.

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are twelve equally weighted questions, each beginning on a new page. Read all eight questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic. **Calculators with stored equations or text are not allowed**. Dictonaries may be used if they have been approved by the proctor before the examination begins. **No other papers or books may be used.**

When you have finished, come to the front of the room and hand your examination paper to the proctor; first put all problems in numerical order and staple them together.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand your exam paper in on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg (0.511 MeV/c^2)
Proton rest mass (m_p)	1.673×10^{-27} kg (938 MeV/c^2)
Neutron rest mass (m_n)	1.675×10^{-27} kg (940 MeV/c^2)
W^+ rest mass (m_W)	80.4 GeV/c^2
Planck's constant (h)	6.63×10^{-34} J$\cdot$ s
Speed of light in vacuum (c)	3.00×10^8 m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Gravitational constant (G)	6.67×10^{-11}N$\cdot$ m^2/kg^2
Permeability of free space (μ_0)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space (ϵ_0)	8.85×10^{-12} F/m
Mass of Earth (M_E)	5.98×10^{24} kg
Equatorial radius of Earth (R_E)	6.38×10^6 m
Avagadro's number (N_A)	6.02×10^{29} m^{-3}
Density of Fe at low temperature (ρ_{Fe})	7.88×10^3 kg/m^3
Classical electron radius (r_0)	2.82×10^{-15} m
Gravitational acceleration on Earth (g)	9.8 m/s^2
Atomic mass unit	1.7×10^{-27} kg
Specific heat of oxygen (c_V)	21.1 J/mole$\cdot$ K
Specific heat of oxygen (c_P)	29.4 J/mole$\cdot$ K

Problem 1

On February 12, 2001, NASA's NEAR-Shoemaker spacecraft landed on the asteroid Eros, a roughly cylindrical shaped object with dimensions $r \approx 7\,\mathrm{km}$, $l \approx 33\,\mathrm{km}$ (see figure). The density of Eros is roughly half the mean density of Earth, or $\rho \approx 3\,\mathrm{g/cm^3}$. The spacecraft's mass is $m \approx 100\,\mathrm{kg}$.

NOTE: Except in part **c.**, you should approximate the cylinder by a sphere of the same volume.

a. What are the spacecraft's mass and weight, respectively, on Earth?
b. Estimate the spacecraft's mass and weight, respectively, on Eros.
c. Will the exact answer to question **b.**, as opposed to your estimate, depend on where on Eros the spacecraft lands? Answer yes or no and qualitatively explain your answer in two (2) sentences.
d. If the spacecraft were just dropped onto Eros from a distance $R \gg l, r$, what would the velocity of impact be?
e. How much energy is needed to decrease the impact velocity to a survivable $5\,\mathrm{km/h}$?

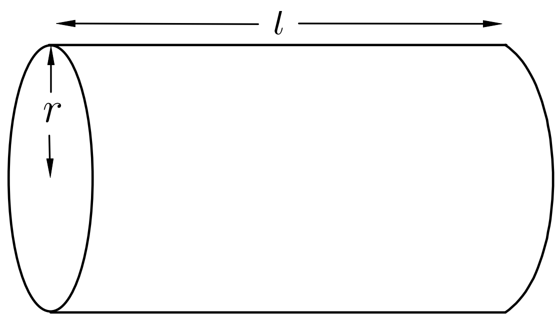

Figure 1: **Figure for Problem 1**

Problem 2

An asteroid of mass m approaches the earth with impact parameter b and relative velocity v_0. Ignore the influence of the Sun and Moon on the asteroid and the Earth.

a. What is the initial angular momentum of the asteroid with respect to the Earth?
b. If the Earth were a point mass, what would the closest approach of the asteroid to the Earth's center be? Express your answer in terms of b, v_0, R_E, the actual radius of the Earth, and g, the acceleration due to gravity on Earth.
c. Will the asteroid hit the Earth? Take $b = 20R_E$, where R_E is radius of the Earth, and $v_0 = 1000\,\mathrm{m/s}$.

Problem 3

The solid homogeneous hemisphere in the figure has a radius R and mass M. It rests on a rough plane surface (so no slipping occurs between the hemisphere and the surface).

a. Calculate the location of the center of mass of the hemisphere.
b. A horizontal force is applied through the center of the flat face of the hemisphere. The force is increased causing the hemisphere to tip. Calculate the force, $F(\theta)$, required to hold the hemisphere in equilibrium at an angle $\theta \leq \pi/2$.
c. At what angle of θ is the hemisphere in unstable equilibrium?

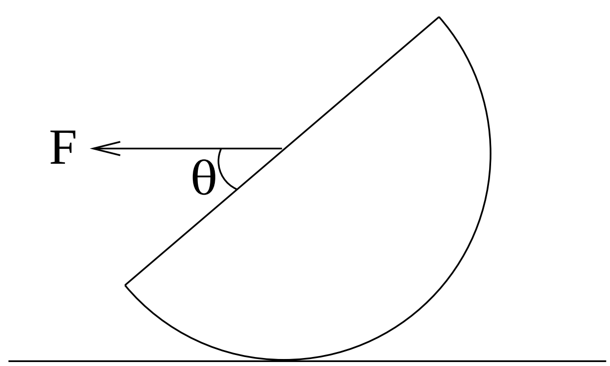

Figure 2: **Figure for Problem 3**

Problem 4

A parallel plate capacitor of plate separation d_1 is filled with a solid dielectric material of permittivity ϵ, as shown in the figure (a) below. The capacitor is charged to voltage V_1. The capacitor is then disconnected from the battery and pulled apart so that the plate separation becomes $d_1 + d_2$. The dielectric does not expand and the dielectric-free region has size d_2 [see figure (b)]. Assuming the plates are large compared to both d_1 and d_2, compute the voltage V_2 after the capacitor is pulled apart.

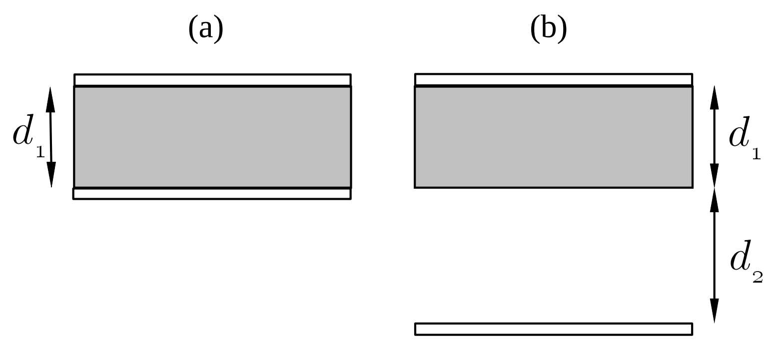

Figure 3: **Figure for Problem 4**

Problem 5

Consider a spherical object of radius R, charged with total charge Q.

a. Calculate the electric field *inside* and *outside* the object for the case of a uniform static distribution of charge.
b. Calculate the electric field *inside* and *outside* the object for the case where the object is a perfect conductor.
c. Sketch the magnitude of the electric field as a function of the radial distance for each case, making your sketches on the figure below.

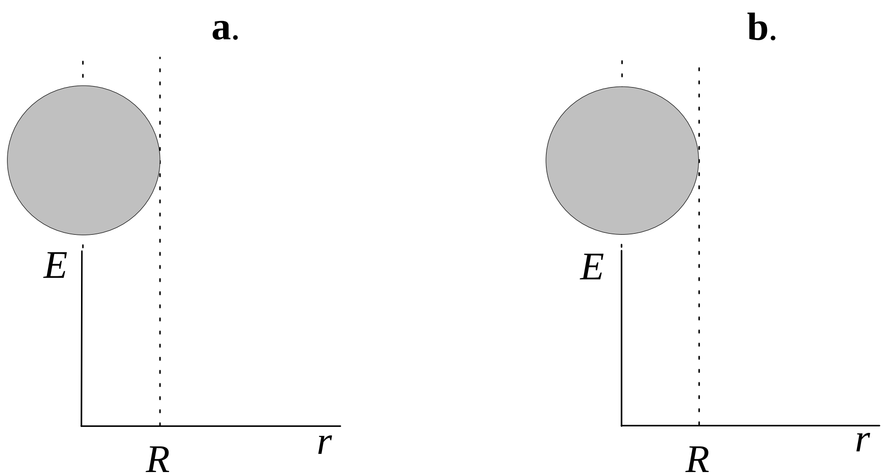

Figure 4: **Figure for Problem 5**

Problem 6

The electric field in a beam of laser light may be approximated by a plane wave $\boldsymbol{E}(\boldsymbol{x}, t) = \hat{x} E_0 \cos(\omega t - kz)$, where $\hat{x}$ is a unit vector and E_0 has a value of $2.4 \times 10^4\,\mathrm{V/m}$. The beam propagates in vacuum and the light has a wavelength of $0.8\,\mu\mathrm{m}$.

a. What is the magnitude and direction of the magnetic field $\boldsymbol{B}(\boldsymbol{x}, t)$?
b. What is the phase of the magnetic field?
c. What is the time-averaged power passing through an area of $1\,\mathrm{cm}^2$ perpendicular to the direction of propagation of the beam?
d. How many photons pass through this area per second?

Problem 7

Consider three particles (denoted p_1, p_2, p_3) each of which may be found in any of three states (denoted s_1, s_2, s_3). No state is favored over any other for occupancy by a particle. There are no interactions between the particles. What is the probability that (any) one of the states will be occupied by two particles

a. when p_1, p_2, and p_3 are all distinguishable from each other?
b. when p_1, p_2, and p_3 are all identical bosons?
c. when p_1 and p_2 are identical bosons, while p_3 is distinguishable from p_1 and p_2?

HINT: Each of the possible configurations of the three particle system is equally likely.

Problem 8

Estimate the height in km at which the atmospheric pressure is half its value at sea level. You may assume the temperature does not vary with height. Make the same estimate using two different approaches:

a. using the condition for hydrostatic equilibrium.
b. using the Maxwell-Boltzmann distribution.

Problem 9

A sample of 5.0 moles of oxygen at temperature 548 K and pressure 1.5 atm occupies a volume of $0.5\,\mathrm{m}^3$. The sample expands adiabatically until its volume doubles. Its pressure is then increased without changing the volume until it returns to the original pressure. Finally, the gas undergoes an isobaric compression until it returns to the original volume.

a. Sketch this cycle on a $P - V$ diagram.
b. What is the change in internal energy during the cycle?
c. What is the net heat added during the cycle?
d. What is the net work done during the cycle?

Problem 10

Consider the following processes. Calculate the threshold *beam* energy for each process.

a. $e^+e^- \to W^+W^-$,
where the electron and positron collide head on with equal energies.

b. $pp \to p\bar{p}pp$,
with one initial proton at rest.

Problem 11

Consider a static observer at the surface of the Earth and a second static observer at a height h above the surface. If the observer at the surface sends a monochromatic light signal of wavelength λ_s to the observer at height h, the light is redshifted to a wavelength λ_h. For a height $h = 220\,\mathrm{m}$, what is the fractional wavelength shift $(\lambda_h - \lambda_s)/\lambda_s$?

Problem 12

Consider a particle of mass m moving in the one-dimensional potential

$$V(x) = \begin{cases} \infty, & x \leq 0, \\ -V_0, & 0 < x < a, \\ 0, & x \geq a. \end{cases}$$

Assume that $V_0 a^2 = 2\pi^2\hbar^2/m$, which allows two energy eigenstates to exist with negative total energy. Let E_1 and E_2 be the energies of these states (you need not determine their values).

a. Determine the wavefunctions $\phi_1(x)$ and $\phi_2(x)$ for $-\infty < x < +\infty$ in terms of the quantities given above, up to their overall normalizations.
b. At time $t = 0$ the particle is prepared in the state $\psi(x,0) = c_1\phi_1(x) + c_2\phi_2(x)$, where c_1 and c_2 are known constants. What is $\psi(x,t)$?
c. Write down an expression for the probability that an energy measurement at time t will give E_2.

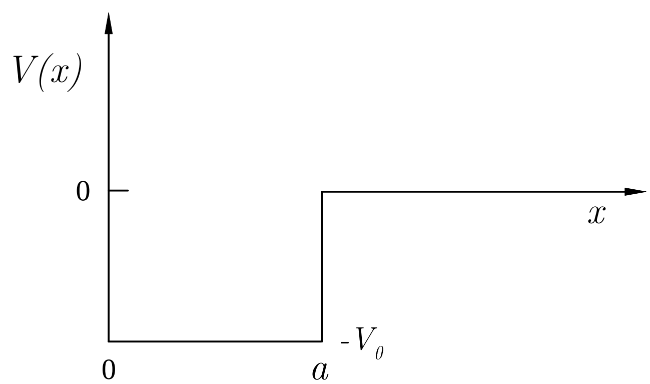

Figure 5: **Figure for Problem 12**