Exam #: ________________________

Printed Name: ________________________

Signature: ________________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Master's Final Examination and Ph.D. Qualifying Examination

PART II

Tuesday, March 31, 2009, 9:00 a.m. to 1:00 p.m.

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are eight equally weighted questions, each beginning on a new page. Read all eight questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. When you start a new problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic and will be provided. **Personal calculators are not allowed**. Dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries are not allowed. No other papers or books may be used.**

When you have finished, come to the front of the room, put all problems in numerical order and staple them together with this sheet on top. Then hand your examination paper to the proctor.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand your exam paper in on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	$1.60 \times 10^{-19}\,\mathrm{C}$
Electron rest mass (m_e)	$9.11 \times 10^{-31}\,\mathrm{kg}$ ($0.511\,\mathrm{MeV/c^2}$)
Proton rest mass (m_p)	$1.673 \times 10^{-27}\,\mathrm{kg}$ ($938\,\mathrm{MeV/c^2}$)
Neutron rest mass (m_n)	$1.675 \times 10^{-27}\,\mathrm{kg}$ ($940\,\mathrm{MeV/c^2}$)
Atomic mass unit	$1.7 \times 10^{-27}\,\mathrm{kg}$
Planck's constant (h)	$6.63 \times 10^{-34}\,\mathrm{J{\cdot}s}$
Speed of light in vacuum (c)	$3.00 \times 10^{8}\,\mathrm{m/s}$
Boltzmann's constant (k_B)	$1.38 \times 10^{-23}\,\mathrm{J/K}$
Gravitational constant (G)	$6.67 \times 10^{-11}\mathrm{N{\cdot}m^2/kg^2}$
Permeability of free space (μ_0)	$4\pi \times 10^{-7}\,\mathrm{H/m}$
Permittivity of free space (ϵ_0)	$8.85 \times 10^{-12}\,\mathrm{F/m}$
Mass of earth (M_E)	$5.98 \times 10^{24}\,\mathrm{kg}$
Equatorial radius of earth (R_E)	$6.38 \times 10^{6}\,\mathrm{m}$
Radius of sun (R_S)	$6.96 \times 10^{8}\,\mathrm{m}$
Classical electron radius (r_0)	$2.82 \times 10^{-15}\,\mathrm{m}$
Specific heat of oxygen (c_V)	$21.1\,\mathrm{J/mole{\cdot}K}$
Specific heat of oxygen (c_P)	$29.4\,\mathrm{J/mole{\cdot}K}$
Specific heat of water ($0^o\,\mathrm{C} < T < 100^0\,\mathrm{C}$)	$4.18\ \mathrm{J/(g{\cdot}K)}$
Latent heat, ice $\rightarrow$ water	$334\,\mathrm{J/g}$
Latent heat, water $\rightarrow$ steam	$2257\,\mathrm{J/g}$
Gravitational acceleration on Earth (g)	$9.8\,\mathrm{m/s^2}$
1 atmosphere	$1.01 \times 10^{5}\ \mathrm{Pa}$

Stirling's Formula

$$\ln(x!) = x\ln(x) - x - \ln\left(\sqrt{2\pi x}\right) + \mathcal{O}(1/x)$$

Integrals

$$\int_{-\infty}^{\infty} dx\, x^{2n}\, e^{-ax^2} = \frac{1 \times 3 \times 5 \times \cdots \times (2n-1)}{2^n\, a^n} \sqrt{\frac{\pi}{a}}$$

$$\int_{0}^{\infty} dx\ e^{-cx} x^n = \frac{n!}{c^{n+1}}$$

Problem 1

A particle of mass m is in a finite spherical well with potential

$$V(r) = \begin{cases} -V_0 & r \leq a \\ 0 & r > a \end{cases},$$

where $V_0 > 0$.

(a) Write down the radial equation for the time independent Schrödinger equation.

(b) What is the angular momentum of the ground state?

(c) Show how to determine the energy of the ground state, assuming that a bound state exists. You should find an equation of the form $f(E) = 0$, where $f(E)$ is a function, such that the ground state energy E is a solution of the equation. You do not need to solve this equation.

(d) For a fixed a, determine the minimum potential depth V_0 for a bound state to exist.

Problem 2

A particle of mass m with energy E is incident from the left on a one dimensional barrier that is approximated by a delta function potential,

$$V(x) = \lambda\,\delta(x) \quad .$$

(a) What is the form of the wave function to the left of $x = 0$? What is the form of the wave function to the right of $x = 0$?

(b) What boundary conditions apply at $x = 0$? (Hint: one way to look at this problem is to use

$$\frac{d}{dx}\,\theta(x > 0) = \delta(x) \quad .$$

where $\theta(x > 0)$ is the function that is 1 for $x > 0$ and 0 for $x < 0$.)

(c) What is the probability $P_{\rm R}$ that the particle will be reflected?

(d) Does the result for $P_{\rm R}$ depend on the sign of λ?

Problem 3

A particle of mass m moving in one dimension is confined by the wedge potential

$$V(x) = \begin{cases} \kappa\, x & x > 0 \\ \infty & x < 0 \end{cases} \,,$$

where $\kappa > 0$.

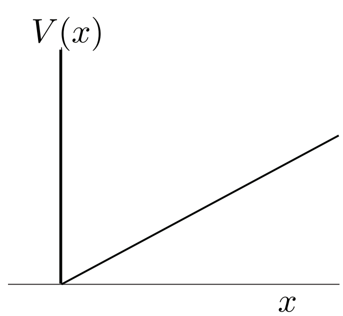
(a) Sketch what the ground state wave function should look like.

(b) Use the variational method to estimate the ground state energy E. For this purpose, use the following variational wave function:

$$\psi(x) = \begin{cases} A\, x\, e^{-ax} & x > 0 \\ 0 & x < 0 \end{cases} \,,$$

Here a is the variational parameter, with $a > 0$, and A is a positive parameter determined from a that adjusts the normalization.

(c) Is the true ground state energy necessarily smaller than the estimated energy that you found? Is the true ground state energy necessarily bigger than the estimated energy that you found? Or could the true ground state energy be either bigger or smaller than the estimated energy that you found?



Problem 4

The Hamiltonian for a particle with spin 1/2 at rest in a magnetic field $\vec{B}$ is given by

$$H = -\gamma\, \vec{B} \cdot \vec{S}$$

where γ is a positive constant and $\vec{S}$ is the spin operator for the particle. Denote the spin state at time t by $\big|\psi(t)\big\rangle$. Suppose that, at time $t = 0$, the spin is in the state

$$\big|\psi(0)\big\rangle = \big|\uparrow\big\rangle \ ,$$

that is, spin up along the z direction.

(a) Suppose the magnetic field is constant as a function of time and is directed along the x-direction,

$$(B_x, B_y, B_z) = (B_0, 0, 0) \ ,$$

with $B_0 > 0$. Find $\big|\psi(t)\big\rangle$ for $t > 0$. Verify that after some time the spin will return to being spin up again, so that

$$\big|\psi(t)\big\rangle = e^{i\phi}\big|\uparrow\big\rangle$$

for some ϕ. Find the value of the phase ϕ corresponding to the first time that this happens after $t = 0$.

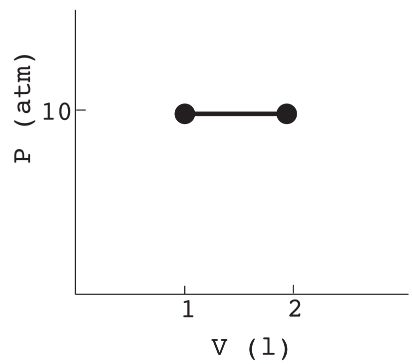
(b) Suppose now that the magnetic field is oscillating in time and is directed along the x-direction:

$$(B_x, B_y, B_z) = (B_0 \sin(\omega t), 0, 0) \ .$$

Find $\big|\psi(t)\big\rangle$ for $t > 0$ for this circumstance. Show that only for values of ω that are sufficiently small will the spin flip to spin down, $\big|\downarrow\big\rangle$, after some time.

Problem 5

One liter of an ideal monoatomic gas, initially at 300^oK, is heated and allowed to expand quasistatically and at a constant pressure of 10 atmospheres to 2 liters, as shown in the P-V diagram below.



(a) Find the work done (in Joules) in this process.

(b) Find the final temperature.

(c) Find the heat added.

(d) Calculate the entropy change of the gas. Is this a reversible process?

Problem 6

Consider N non-interacting classical spins on a lattice. Each spin can point either up or down and has a magnetic moment μ. The system is subject to a uniform magnetic field B, and is in thermal equilibrium at a temperature T.

(a) Find an expression for the mean energy of magnetization of the system.

(b) Find an expression for the variance (i.e., the square of the standard deviation) of the energy of magnetization.

(c) Explain in words why the behavior of your results in the limits $T \to 0$ and $T \to \infty$ makes sense.

Problem 7

This problem concerns phase changes of water at normal atmospheric pressure.

(a) 100 g of water at 100^{o}C is placed in thermal contact with 1 kg of ice at 0^{o}C and the system is allowed to come to equilibrium. Find the equilibrium temperature. Calculate the entropy change of the original 100 g of water, of the original 1 kg of ice, and of the system as a whole.

(b) 1 kg of water at 100^{o}C is placed in thermal contact with 1 kg of ice at 0^{o}C, and the system is allowed to come to equilibrium. Again, find the equilibrium temperature, and calculate the entropy change of the original 1 kg of water, of the original 1 kg of ice, and of the system as a whole.

Problem 8

A large number N of spin 0, non-interacting Bosons move in a one dimensional box of length L. There is an attractive potential at the center of the box that creates a single one-particle bound state of energy $\epsilon_0 < 0$. The potential has negligible effect on the other (free particle) states of the system.

(a) What are the allowed energies of the single particle states of this system?

(b) What are the occupation numbers of each of these states if the chemical potential is μ?

(c) What is the upper limit on μ?

(d) Show that this upper limit on μ implies an upper limit $\rho_{max}(T)$ on the density $\rho_{excited} \equiv \langle N_{excited} \rangle / L$, where $\langle N_{excited} \rangle$ is the mean total number of particles in *all* of the excited states. (Your expression for $\rho_{max}(T)$ may involve an integral, which you need not evaluate for this part of the problem, but you must show that this integral is actually finite).

(e) For all temperatures $T < T_c$, where T_c is the Bose-Einstein condensation temperature, $\rho_{max}(T) < N/L \equiv \rho$, and the ground state becomes macroscopically occupied. Assuming that $k_B T_c \ll |\epsilon_0|$, find an equation of the form $f(k_B T_c, \rho) = 0$ satisfied by T_c. You need not solve this equation, but your expression for $f(k_B T_c, \rho)$ must be explicit (i.e., it must not involve any integrals, sums, etc.).