Exam #: ______________________

Printed Name: ______________________

Signature: ______________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Ph.D. Qualifying Examination
and
Master's Final Examination, PART II

Tuesday, April 1, 2008, 1:00 p.m. to 5:00 p.m.

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are eight equally weighted questions, each beginning on a new page. Read all eight questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic, and will be provided. **Personal calculators of any type are not allowed**. Paper dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries will not be allowed. No other papers or books may be used.**

When you have finished, come to the front of the room and hand your examination paper to the proctor; first put all problems in numerical order and staple them together.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand in your exam paper on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	$1.60 \times 10^{-19}\,\mathrm{C}$
Electron rest mass (m_e)	$9.11 \times 10^{-31}\,\mathrm{kg}$ ($0.511\,\mathrm{MeV/c^2}$)
Proton rest mass (m_p)	$1.673 \times 10^{-27}\,\mathrm{kg}$ ($938\,\mathrm{MeV/c^2}$)
Neutron rest mass (m_n)	$1.675 \times 10^{-27}\,\mathrm{kg}$ ($940\,\mathrm{MeV/c^2}$)
W^+ rest mass (m_W)	$80.4\,\mathrm{GeV/c^2}$
Planck's constant (h)	$6.63 \times 10^{-34}\,\mathrm{J \cdot s}$
Speed of light in vacuum (c)	$3.00 \times 10^{8}\,\mathrm{m/s}$
Boltzmann's constant (k_B)	$1.38 \times 10^{-23}\,\mathrm{J/K}$
Gravitational constant (G)	$6.67 \times 10^{-11}\mathrm{N \cdot m^2/kg^2}$
Permeability of free space (μ_0)	$4\pi \times 10^{-7}\,\mathrm{H/m}$
Permittivity of free space (ϵ_0)	$8.85 \times 10^{-12}\,\mathrm{F/m}$
Bohr magneton (μ_B)	$9.27 \times 10^{-24}\,\mathrm{J/T}$
Mass of Earth (M_{Earth})	$5.98 \times 10^{24}\,\mathrm{kg}$
Mass of Moon (M_{Moon})	$7.35 \times 10^{22}\,\mathrm{kg}$
Mass of Sun (M_{Sun})	$1.99 \times 10^{30}\,\mathrm{kg}$
Radius of Earth (R_{Earth})	$6.38 \times 10^{6}\,\mathrm{m}$
Radius of Moon (M_{Moon})	$1.74 \times 10^{6}\,\mathrm{m}$
Radius of Sun (R_{Sun})	$6.96 \times 10^{8}\,\mathrm{m}$
Earth - Sun distance (R_{ES})	$1.50 \times 10^{11}\,\mathrm{m}$
Classical electron radius (r_0)	$2.82 \times 10^{-15}\,\mathrm{m}$
Gravitational acceleration on Earth (g)	$9.8\,\mathrm{m/s^2}$
Atomic mass unit	$1.66 \times 10^{-27}\,\mathrm{kg}$
One atmosphere ($1\,\mathrm{atm}$)	$1.01 \times 10^{5}\,\mathrm{N/m^2}$

Integrals

$$\int_0^\infty e^{-a^2x^2}dx = \frac{\sqrt{\pi}}{2a} \qquad \text{(i)}$$

$$\int_0^\infty xe^{-x^2}dx = \frac{1}{2} \qquad \text{(ii)}$$

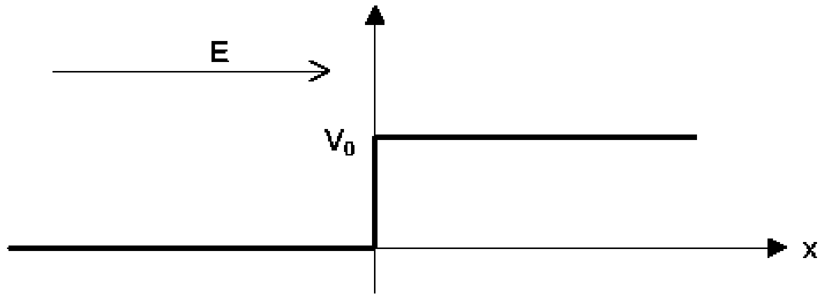
$$\int_0^\infty x^2e^{-x^2}dx = \frac{\sqrt{\pi}}{4} \qquad \text{(iii)}$$

$$\int_0^\infty x^{2n}e^{-x^2}dx = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^{n+1}a^n}\sqrt{\frac{\pi}{a}} \qquad \text{(iv)}$$

$$\int_0^\infty x^n e^{-ax}dx = \frac{\Gamma(n+1)}{a^{n+1}},\ n > -1,\ a > 0 \qquad \text{(v)}$$

Problem 1

A particle of mass m and energy E approaches a one dimensional step potential of height $V_\circ$ beginning at x = 0 as shown in the figure.



(a) Define the reflection coefficient R and the transmission coefficient T.

(b) For $E > V_\circ$ determine these reflection and transmission coefficients.

(c) If the incident wave amplitude is unity, will R + T = 1? Why or why not?

Problem 2

The muon lifetime is 2.2 microseconds, and its mass is 106 MeV/c^2. Muons are created in the atmosphere by cosmic rays, and arrive on the surface of the Earth. A typical atmospheric muon energy is 1 GeV (= 1000 MeV).

(a) Calculate the distance a 1 GeV muon will travel in 2.2 microseconds according to an Earthly observer.

(b) Calculate the fraction of 1 GeV muons that will be left undecayed after 2.2 microseconds according to an Earthly observer, taking into account the time dilation effect.

(c) The Heisenberg uncertainty relation says an unstable particle has an uncertainty in its energy due to its finite period of existence. Estimate the relative uncertainty in the energy of the 1 GeV muon. Show that this is a negligible effect. Does this relative uncertainty depend on the frame of reference?

Problem 3

A particle of mass m is moving in one-dimension according to the Hamiltonian

$$H = \frac{p^2}{2m} + \gamma x^2, \tag{1}$$

where p is the momentum operator, x is the position operator, and γ is a constant with dimensions of mass/(time)^2.

(a) The ground state wavefunction is given by:

$$\psi_\circ = (2\alpha/\pi)^{1/4}\, e^{-\alpha x^2}, \tag{2}$$

where $\alpha = \sqrt{m\gamma/(2\hbar^2)}$. Determine the ground state energy in terms of m, γ, and Planck's constant $\hbar$.

For parts (b) and (c) assume that the particle has charge q and is now subjected to an electric field, $\mathcal{E}$, making the Hamiltonian:

$$H' = \frac{p^2}{2m} + \gamma x^2 - q\mathcal{E}x. \tag{3}$$

(b) Consider the electrostatic term in H' as a small perturbation to H. Show that the first order correction to the ground state energy is zero.

(c) An exact solution to the ground state energy under H' can be found by rewriting H' in terms of a displaced position $\tilde{x} = x - x_\circ$, for some $x_\circ$, so that

$$\tilde{H} = \frac{\tilde{p}^2}{2m} + A\tilde{x}^2 + C, \tag{4}$$

where A and C are constants and $\tilde{p} = p$. Do this, and thus determine the exact ground state energy in terms of m, γ, q, $\mathcal{E}$ and $\hbar$.

Problem 4

Consider the operator X which (in matrix form) is given by:

$$\mathrm{X} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \tag{5}$$

(a) Calculate its eigenvalues and eigenvectors.

(b) If a state (1,1,1) is passed through a device which measures X, what is the probability of the outcome "1"?

Problem 5

The function $\mathcal{H} = \mathrm{U} + \mathrm{PV}$ has been found to be useful for the study of many thermodynamic processes.

(a) What is the function $\mathcal{H}$ called?

(b) Show that $\mathcal{H}$ is conserved when a gas is slowly transferred from one thermally isolated reservoir at a given pressure to another at a lower pressure through a partition containing microscopic pores (the Joule-Thomson Process).

(c) One mole of an ideal gas is passed through a porous plug from pressure $\mathrm{P}_1 = 1$ MPa to pressure $\mathrm{P}_2 = 100$ kPa in the Joule-Thomson process. Calculate the change in the entropy of the gas during this process.

Problem 6

Hydrogen has the approximate energy spectrum

$$E_n = -\frac{\mathcal{R}}{n^2} \tag{6}$$

where $\mathcal{R}$ is the Rydberg constant, and $n = 1, 2, 3, ...$ is the principal quantum number. Each level E_n has degeneracy of $g_n = 2n^2$. Neglect all states with $n > 3$, and assume that we are able to study isolated atoms in thermal equilibrium at absolute temperature T, using classical statistics.

(a) Find an expression for the probability P_n of finding a hydrogen atom in level n at the temperature T.

(b) Assume the temperature T is such that $P_2/P_1 = 0.01$ under the approximations stated above. What is the ratio between $\mathcal{R}$ and $k_B T$ at that point? Here, k_B is the Boltzmann constant.

Problem 7

Consider N atoms in a row. Each atom can exist in only one of two states, a "down" state with energy 0 and an "up" state with energy $\epsilon > 0$. Atom number j can only be in an "up" state if all the atoms 1 to j-1 are also "up"–like a zipper in which a link can be open only if those to the left are all open. The system is at absolute temperature T.

(a) For $N = 4$, sketch all the possible microstates of the system and indicate their energies.

(b) Now consider arbitrary N. Write the partition function for the system. Evaluate any sums that appear in your expression. You may make use of the following for a truncated geometric series,

$$\sum_{j=0}^{N} r^j = \frac{1 - r^{N+1}}{1 - r} \tag{7}$$

(c) Continuing part (b), write an expression for $\langle s \rangle$, the expected number of "up" atoms you would encounter as a function of T. You may leave your answer in the form of a sum, if you wish.

(d) Derive an approximate expression for $\langle s \rangle$ in the low temperature regime, $\epsilon \gg k_B T$, where k_B is the Boltzmann constant. "Zero" is not an acceptable answer – consider the lowest nonzero function of temperature. Your answer should not involve any unevaluated sums.

(e) Consider the case $N = 2$ in the high temperature limit $\epsilon / k_B T \to 0$. What is $\langle s \rangle$?

Problem 8

For an ideal gas in thermal equilibrium, the distribution of speeds of the gas molecules is given by the Maxwell-Boltzmann distribution

$$N_v dv = 4\pi N \left(\frac{m}{2\pi k_B T} \right)^{3/2} v^2 e^{-mv^2/2k_B T} dv \tag{8}$$

where N_v is the number of molecules with speed between v and $v + dv$, N is the total number of molecules, m is the mass of one molecule, k_B is the Boltzmann constant, and T is the absolute temperature. Determine the average (mean) speed, the root-mean-square speed, and the most likely speed of the gas molecules.