Exam #: ______________________

Printed Name: ______________________

Signature: ______________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Ph.D. Qualifying Examination, PART II

Tuesday, April 3, 2007, 1:00 p.m. to 5:00 p.m.

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are six equally weighted questions, each beginning on a new page. Read all six questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic, and will be provided. **Personal calculators of any type are not allowed**. Paper dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries will not be allowed. No other papers or books may be used.**

When you have finished, come to the front of the room and hand your examination paper to the proctor; first put all problems in numerical order and staple them together.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand your exam paper in on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	$1.60 \times 10^{-19}\,\mathrm{C}$
Electron rest mass (m_e)	$9.11 \times 10^{-31}\,\mathrm{kg}$ ($0.511\,\mathrm{MeV/c^2}$)
Proton rest mass (m_p)	$1.673 \times 10^{-27}\,\mathrm{kg}$ ($938\,\mathrm{MeV/c^2}$)
Neutron rest mass (m_n)	$1.675 \times 10^{-27}\,\mathrm{kg}$ ($940\,\mathrm{MeV/c^2}$)
W^+ rest mass (m_W)	$80.4\,\mathrm{GeV/c^2}$
Planck's constant (h)	$6.63 \times 10^{-34}\,\mathrm{J\cdot s}$
Speed of light in vacuum (c)	$3.00 \times 10^{8}\,\mathrm{m/s}$
Boltzmann's constant (k_B)	$1.38 \times 10^{-23}\,\mathrm{J/K}$
Gravitational constant (G)	$6.67 \times 10^{-11}\mathrm{N\cdot m^2/kg^2}$
Permeability of free space (μ_0)	$4\pi \times 10^{-7}\,\mathrm{H/m}$
Permittivity of free space (ϵ_0)	$8.85 \times 10^{-12}\,\mathrm{F/m}$
Mass of Earth (M_{Earth})	$5.98 \times 10^{24}\,\mathrm{kg}$
Mass of Moon (M_{Moon})	$7.35 \times 10^{22}\,\mathrm{kg}$
Radius of Earth (R_{Earth})	$6.38 \times 10^{6}\,\mathrm{m}$
Radius of Moon (M_{Moon})	$1.74 \times 10^{6}\,\mathrm{m}$
Radius of Sun (R_{Sun})	$6.96 \times 10^{8}\,\mathrm{m}$
Mercury - Sun distance (R_{MS})	$5.79 \times 10^{10}, \mathrm{m}$
Earth - Sun distance (R_{ES})	$1.50 \times 10^{11}\,\mathrm{m}$
Density of iron at low temperature (ρ_{Fe})	$7.88 \times 10^{3}\,\mathrm{kg/m^3}$
Classical electron radius (r_0)	$2.82 \times 10^{-15}\,\mathrm{m}$
Gravitational acceleration on Earth (g)	$9.8\,\mathrm{m/s^2}$
Atomic mass unit	$1.66 \times 10^{-27}\,\mathrm{kg}$

Spherical harmonics

The spherical harmonics Y_{lm} have the normalization property

$$\int_{-1}^{1} d\cos\theta \int_{0}^{2\pi} d\phi \; Y^{\star}_{lm}(\theta,\phi)\, Y_{l'm'}(\theta,\phi) = \delta_{ll'}\,\delta_{mm'}.$$

The first few are

$$\begin{aligned}
Y_{00} &= \frac{1}{\sqrt{4\pi}} \\
Y_{11} &= -\sqrt{\frac{3}{8\pi}}\,\sin\theta\, e^{i\phi} \\
Y_{10} &= \sqrt{\frac{3}{4\pi}}\,\cos\theta \\
Y_{22} &= \sqrt{\frac{15}{32\pi}}\,\sin^2\theta\, e^{2i\phi} \\
Y_{21} &= -\sqrt{\frac{15}{8\pi}}\,\sin\theta\,\cos\theta\, e^{i\phi} \\
Y_{20} &= \sqrt{\frac{5}{16\pi}}\,[3\cos^2\theta - 1]
\end{aligned}$$

with $Y_{l,-m}(\theta,\phi) = -Y^{*}_{lm}(\theta,\phi)$.

Laplacian operator

Cartesian coordinates:

$$\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

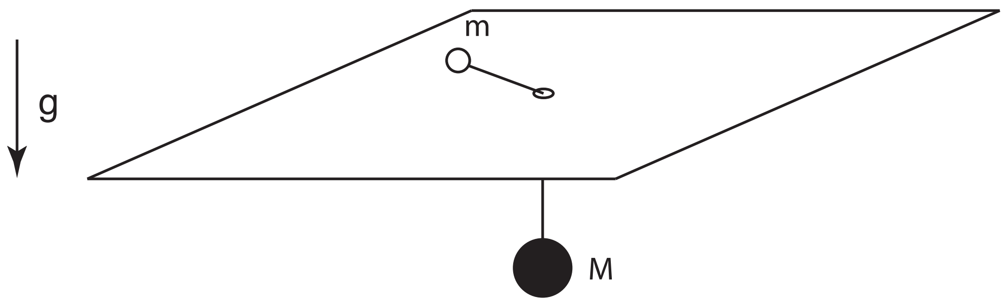
Cylindrical coordinates:

$$\nabla^2 t = \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial t}{\partial r}) + \frac{1}{r^2}\frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}$$

Spherical coordinates:

$$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial t}{\partial r}) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial t}{\partial\theta}) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$

Problem 1



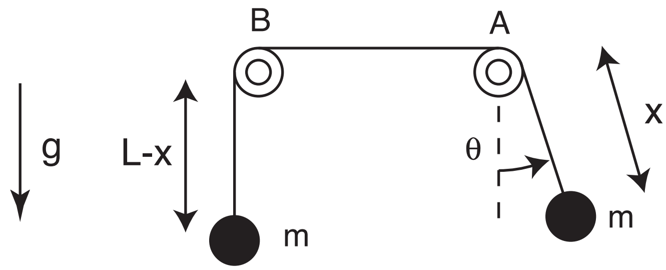
Consider a mass m sliding without rolling or friction on a flat, horizontal, and perfectly smooth table with a small hole. An inextensible string passes through the hole and connects the mass m to a second mass M which hangs suspended vertically under the influence of gravity. The hole is small enough that the mass M can only move in the vertical direction.

a) Write the Lagrangian for this system in terms of the two generalized coordinates r and ϕ, where r is the radial distance of mass m from the hole and ϕ is the azimuthal angle of m in the plane of the table.

b) Identify two conservation laws involving $\dot{\phi}$ and/or $\dot{r}$ and use them to solve for the generalized velocities $\dot{r}$ and $\dot{\phi}$ *as functions of the radial coordinate* r.

c) If the mass M is initially at rest, and the mass m is initially moving at speed v_0, find the minimum and maximum values of r.

Problem 2



Two weights, each with mass m, are connected with a massless string and suspended from two massless, frictionless pulleys in a gravitational field as shown above. Denote the length of string from pulley A to mass A by x. The length of string from pulley B to mass B is $L - x$, where L is fixed by the geometry of the system. The radius of each pulley is small enough compared to x and $L - x$ that it can be approximated by zero.

a) Suppose that each mass hangs straight down, but that the masses can move up or down. What is the Lagrangian for this system? What is the equation of motion for $x(t)$? If the system starts with initial conditions $x(0) = x_0$, $\dot{x}(0) = 0$, what is $x(t)$ for later times?

b) Now consider what happens if mass A is allowed to swing in the plane of the drawing. Denote the angle that string A makes with the vertical direction by θ. Using the two coordinates x and θ, what is the Lagrangian for this system? What are the equations of motion?

c) To solve the equations of motion, we will consider a limiting case. Suppose one draws mass A to the side and releases if from rest, such that the system starts from $\theta(0) = \theta_0 \ll 1$ with $\dot{\theta}(0) = 0$ and $x(0) = x_0, \dot{x}(0) = 0$. As long as $\dot{x}(t)$ remains small, we can approximate $x(t) \approx x_0$ and $\dot{x}(t) \approx 0$ in the equation of motion for θ. Then use the solution for $\theta(t)$ in the equation of motion for x. Using this approximation, calculate $\langle \ddot{x}(t) \rangle$ (averaging over the oscillations in θ) and determine if mass A accelerates up, down, oscillates, or remains stationary.

Problem 3

Due to distortions from spherical symmetry, the sun's gravitational potential in the equatorial plane acquires a "quadrupole term"

$$\delta U(r) = -\frac{\epsilon G M m R^2}{3r^3}$$

in addition to the gravitational potential if the sun were perfectly spherical. Here, G is the gravitational constant, M and R are the sun's mass and radius respectively, m and r are the mass and distance from the center of the sun of a small planet moving in the equatorial plane (e.g.: Mercury), and ϵ is a dimensionless number ($\epsilon \ll 1$).

a) Find the period of a circular orbit of radius r.

b) Find the frequency of radial oscillations about a nearly circular orbit of radius r.

c) Calculate the rate of precession of the perihelion of a nearly circular orbit.

d) If the period of Mercury's nearly circular orbit is 88 days, and the perihelion of Mercury's orbit precesses by 43 arc-seconds per 100 years, find the value of ϵ which would fully explain this precession classically from the additional quadrupole term alone.

Problem 4

A classical model for a nonpolar molecule is a damped harmonic oscillator. Here, consider a charge $-q$ with mass m attached to another charge of $+q$ with infinite mass by a harmonic "spring" with spring constant C and damping constant γ. The equilibrium separation of the charges is zero if no external forces are applied.

a) When the molecule is driven by monochromatic light with frequency ω and amplitude $E_\circ$, determine the steady-state amplitude of the oscillating dipole moment induced by the light.

b) For a dilute molecular vapor with density N, determine the electric susceptibility χ of the vapor (including only contributions related to the spring).

c) Using the result from b), plot schematically the real part of the index of refraction n_R as a function of ω near the molecular absorption resonance $\omega_\circ$. Note: $n = \sqrt{1+\chi} \approx 1 + \frac{1}{2}\chi$ for small χ. χ is small for this problem.

d) Can the group velocity ever become greater than the speed of light? Comment.

Problem 5

A particle of charge $q > 0$ and mass m is placed and held at the origin of a region with magnetic field, $\vec{B} = B_\circ \hat{x}$, and electric field, $\vec{E} = E_\circ \hat{z}$, where E_0 and B_0 are constants. At $t = 0$, the charge is released.

a) Find $x(t)$, $y(t)$, and $z(t)$ for the charge after it is released.

b) Show that the trajectory represents the position of a point on the rim of a wheel, radius $R = qE_\circ/(m\omega^2)$, rolling along the y-axis at speed $E_\circ/B_\circ$. Here, ω is the cyclotron frequency.

c) Using the approximation $\cos(\omega t) \approx 1\text{-}\frac{1}{2}(\omega t)^2$, valid for $(\omega t)^2 \ll 1$, evaluate $z(t)$ and $\dot{z}(t)$ to order $\mathcal{O}[(\omega t)^2]$. Why does $y(t)$ vanish to the same order?

d) If the charge is given an appropriate initial velocity, it will move in a straight line parallel to the y-axis. Find this velocity.

Problem 6

Consider an infinite dielectric medium, with permittivity $\epsilon = \epsilon_r \epsilon_\circ$, with a spherical cavity of radius R cut out from it. The hollow space has permittivity $\epsilon_\circ$. Located at the center of the cavity is a point charge q. There are no other free charges.

a) Write down the boundary conditions for the electric potential $\Phi(\vec{r})$ and displacement field $\vec{D}(\vec{r})$ at the interface between the dielectric material and the cavity.

b) Determine the potential inside and outside the cavity.

c) Determine the charge density σ induced on the surface of the cavity.

d) If we could remove the free charge q from the center of the cavity but keep the charge density on the surface of the cavity, what would be the potential and electric field inside the cavity?