Exam #: ______________________

Printed Name: ______________________

Signature: ______________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Ph.D. Qualifying Examination, PART II

Tuesday, April 4, 2006, 1:00 p.m. to 5:00 p.m.

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are six equally weighted questions, each beginning on a new page. Read all six questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

You will not need calculators for this exam. **Calculators or any other electronic devices are not allowed, including electronic dictionaries**. Paper dictionaries may be used if they have been approved by the proctor before the examination begins. **No other papers or books may be used.**

When you have finished, come to the front of the room and hand your examination paper to the proctor; first put all problems in numerical order and staple them together.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand your exam paper in on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	$1.60 \times 10^{-19}\,\mathrm{C}$
Electron rest mass (m_e)	$9.11 \times 10^{-31}\,\mathrm{kg}$ ($0.511\,\mathrm{MeV/c^2}$)
Proton rest mass (m_p)	$1.673 \times 10^{-27}\,\mathrm{kg}$ ($938\,\mathrm{MeV/c^2}$)
Neutron rest mass (m_n)	$1.675 \times 10^{-27}\,\mathrm{kg}$ ($940\,\mathrm{MeV/c^2}$)
W^+ rest mass (m_W)	$80.4\,\mathrm{GeV/c^2}$
Planck's constant (h)	$6.63 \times 10^{-34}\,\mathrm{J \cdot s}$
Speed of light in vacuum (c)	$3.00 \times 10^{8}\,\mathrm{m/s}$
Boltzmann's constant (k_B)	$1.38 \times 10^{-23}\,\mathrm{J/K}$
Gravitational constant (G)	$6.67 \times 10^{-11}\mathrm{N \cdot m^2/kg^2}$
Permeability of free space (μ_0)	$4\pi \times 10^{-7}\,\mathrm{H/m}$
Permittivity of free space (ϵ_0)	$8.85 \times 10^{-12}\,\mathrm{F/m}$
Mass of Earth (M_{Earth})	$5.98 \times 10^{24}\,\mathrm{kg}$
Mass of Moon (M_{Moon})	$7.35 \times 10^{22}\,\mathrm{kg}$
Radius of Earth (R_{Earth})	$6.38 \times 10^{6}\,\mathrm{m}$
Radius of Moon (M_{Moon})	$1.74 \times 10^{6}\,\mathrm{m}$
Radius of Sun (R_{Sun})	$6.96 \times 10^{8}\,\mathrm{m}$
Earth - Sun distance (R_{ES})	$1.50 \times 10^{11}\,\mathrm{m}$
Density of iron at low temperature (ρ_{Fe})	$7.88 \times 10^{3}\,\mathrm{kg/m^3}$
Classical electron radius (r_0)	$2.82 \times 10^{-15}\,\mathrm{m}$
Gravitational acceleration on Earth (g)	$9.8\,\mathrm{m/s^2}$
Atomic mass unit	$1.66 \times 10^{-27}\,\mathrm{kg}$
Specific heat of oxygen (c_V)	$21.1\,\mathrm{J/mole \cdot K}$
Specific heat of oxygen (c_P)	$29.4\,\mathrm{J/mole \cdot K}$

Spherical harmonics

The spherical harmonics Y_{lm} have the normalization property

$$\int_{-1}^{1} d\cos\theta \int_0^{2\pi} d\phi \; Y_{lm}^{\star}(\theta,\phi)\, Y_{l'm'}(\theta,\phi) = \delta_{ll'}\,\delta_{mm'}.$$

The first few are

$$\begin{aligned}
Y_{00} &= \frac{1}{\sqrt{4\pi}} \\
Y_{11} &= -\sqrt{\frac{3}{8\pi}}\,\sin\theta\, e^{i\phi} \\
Y_{10} &= \sqrt{\frac{3}{4\pi}}\,\cos\theta \\
Y_{22} &= \sqrt{\frac{15}{32\pi}}\,\sin^2\theta\, e^{2i\phi} \\
Y_{21} &= -\sqrt{\frac{15}{8\pi}}\,\sin\theta\,\cos\theta\, e^{i\phi} \\
Y_{20} &= \sqrt{\frac{5}{16\pi}}\,[3\cos^2\theta - 1]
\end{aligned}$$

with $Y_{l,-m}(\theta,\phi) = -Y^{*}_{lm}(\theta,\phi)$.

Laplacian operator

Cartesian coordinates:

$$\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

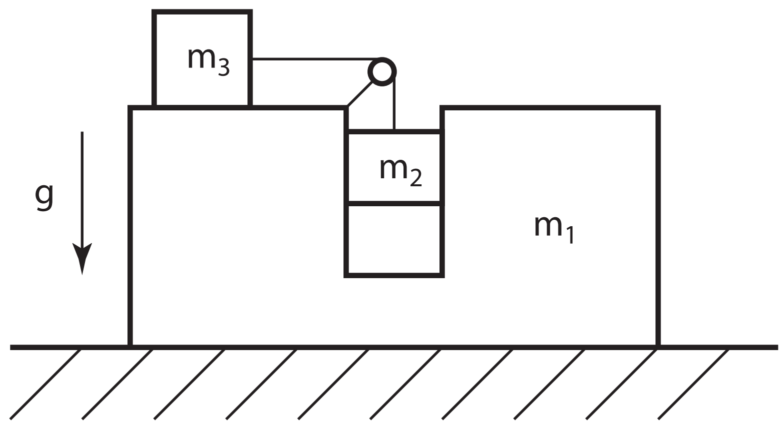
Cylindrical coordinates:

$$\nabla^2 t = \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial t}{\partial r}) + \frac{1}{r^2}\frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}$$

Spherical coordinates:

$$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial t}{\partial r}) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial t}{\partial\theta}) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$

Problem 1



Consider the "pedagogical machine" shown above. Gravity acts vertically with acceleration g. The mass m_2 slides vertically and the mass m_3 slides horizontally, connected by a massless cord across a pulley. The entire machine can slide horizontally on a table. All motion is assumed to be frictionless.

a) Write down the Lagrangian for this system. You do not need to worry about what happens when m_2 reaches the bottom of the slot.

b) Find the acceleration (magnitude and direction) of the mass m_1.

c) Describe exactly where the unbalanced force is acting on m_1.

Problem 2

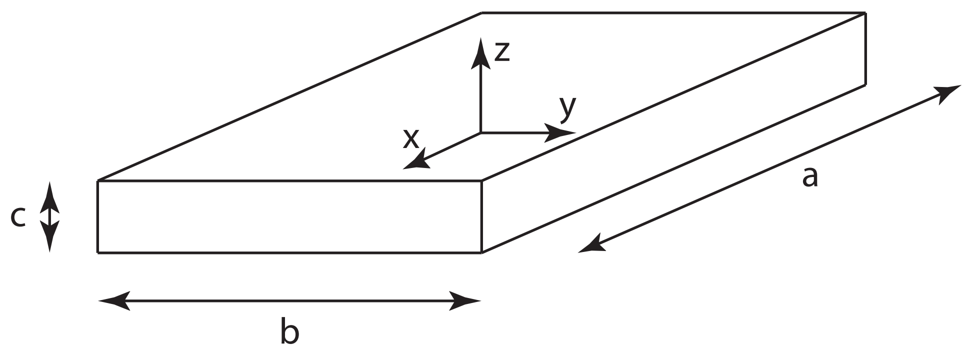
The air in a large hot air balloon is heated until it just hovers above the ground. Assume that the total mass of the balloon and it's payload is M. At time $t = 0$, a sandbag with initial mass m_0 ($m_0 \ll M$) is emptied at a constant rate, until a time $t = T$ when the sandbag is completely empty. Neglect air resistance and assume the buoyancy of the balloon is constant.

a) Find the velocity of the balloon at time $t = T$.

b) Find the height of the balloon at time $t = T$ to second order in m_0/M.

Problem 3

Consider the rigid body rotations of a uniform rectangular block with dimensions a, b, and c such that $a > b > c$, as shown in the figure below. The origin of the coordinate system is at the center of mass of the block. We wish to show that the block has axes of stable rotation parallel to the longest and shortest sides, but rotations about the third axis are not stable. (A stable rotation axis is one for which the direction of the angular velocity is stable to small, orthogonal components of angular velocity.) There are no external forces.



a) Identify the principal axes $\hat{e}_1$, $\hat{e}_2$, and $\hat{e}_3$ in terms of the given coordinate system, such that the moments of inertia satisfy $I_3 > I_2 > I_1$.

b) Start with the Euler equation for the torque $\vec{\tau}$,

$$\vec{\tau} = \frac{d'\vec{L}}{dt} + \vec{\omega} \times \vec{L},$$

where $\vec{L} = \mathbf{I} \cdot \vec{\omega}$ is the angular momentum and the derivative is with respect to the rotating system. Show that this leads to three equations of the form:

$$\begin{aligned}
\ddot{\omega}_1 + K_1\omega_1 &= 0 \\
\ddot{\omega}_2 + K_2\omega_2 &= 0 \\
\ddot{\omega}_3 + K_3\omega_3 &= 0
\end{aligned}$$

where K_1 is a function of ω_2, ω_3, and the moments of inertia, and analogously for K_2 and K_3.

c) For rotations nearly along $\hat{e}_3$, such that $\omega_3 \gg \omega_1$ and $\omega_3 \gg \omega_2$, show that the first two equations above imply that such rotations are stable. Show that a similar conclusion can be made for rotations nearly along $\hat{e}_1$, but rotations nearly along $\hat{e}_2$ are *not* stable.

Problem 4

Consider a homogenous, isotropic medium with real-valued electric permittivity ε and magnetic permeability μ. In parts a) and b) below, we specifiy two different types of electromagnetic field, $\vec{E}$ and $\vec{H}$. For each of these cases below, calculate the energy density and Poynting vector.

The cases are as follows:

a) In Cartesian coordinates x, y, z:

$$\vec{E}(\vec{r}, t) = E_0 \hat{y} e^{i(kx - \omega t)}.$$

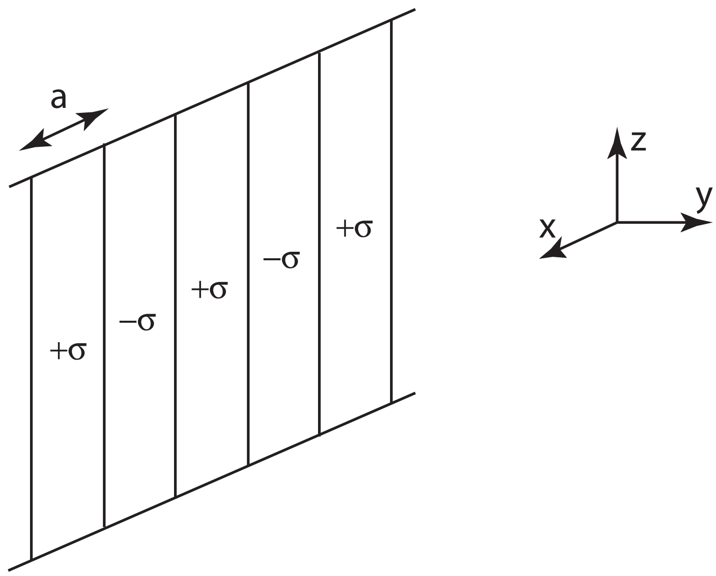
Here $k = \sqrt{\varepsilon\mu}\,\omega$ is real. Find $\vec{H}(\vec{r}, t)$ from Maxwell's equations.

b) Same as in a), but with $k = i\kappa$, where $\kappa > 0$ is real, so that k is now imaginary.

c) Explain why the field in a) can transport momentum to $x \to \infty$, whereas the field in b) cannot.

Problem 5

Consider a planar charge distribution in the $y = 0$ plane for which the surface charge density runs in infinite strips parallel to the z-axis. Each strip has a width a with constant charge density across the strip. The sign of the surface charge density is opposite on neighboring strips, such that every other strip has a charge density of $+\sigma$, while the remaining strips have a charge density of $-\sigma$. Find the resulting electric potential away from the plane.



Hint: you may wish to start by considering the region near one of the $+\sigma$ strips. Close the the surface, the strip will resemble an infinite plane.

Problem 6

Suppose the electric field in vacuum of a point charge q is actually $\vec{E} = q\hat{r}/r^{2+\delta}$ where $\delta \ll 1$, rather than $\vec{E} = q\hat{r}/r^2$.

a) For this field, calculate $\vec{\nabla} \cdot \vec{E}$ and $\vec{\nabla} \times \vec{E}$ for $r > 0$.

b) Find the electric potential for such a point charge.

c) Two concentric spherical conduction shells of radii a and b are joined by a thin conducting wire. Show that if charge Q_a resides on the outer shell, then the charge on the inner shell would be given by

$$Q_b = -\frac{Q_a \delta}{2(a-b)} \left[2b \ln 2a - (a+b)\ln(a+b) + (a-b)\ln(a-b)\right]$$

Hints for c):

For small δ, $x^{1-\delta} \approx x(1 - \delta \ln x)$ and $1 - \delta^2 \approx 1$.

The integral in this calculation can be simplified using $\sin\theta d\theta = d(\cos\theta)$.