Exam #: ______________________

Printed Name: ______________________

Signature: ______________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Ph.D. Qualifying Examination, PART II

Tuesday, March 29, 2005, 1:00 p.m. to 5:00 p.m.

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are six equally weighted questions, each beginning on a new page. Read all six questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic. **Calculators with stored equations or text are not allowed**. Dictionaries may be used if they have been approved by the proctor before the examination begins. **No other papers or books may be used.**

When you have finished, come to the front of the room and hand your examination paper to the proctor; first put all problems in numerical order and staple them together.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand your exam paper in on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	$1.60 \times 10^{-19}\,\mathrm{C}$
Electron rest mass (m_e)	$9.11 \times 10^{-31}\,\mathrm{kg}$ ($0.511\,\mathrm{MeV/c^2}$)
Proton rest mass (m_p)	$1.673 \times 10^{-27}\,\mathrm{kg}$ ($938\,\mathrm{MeV/c^2}$)
Neutron rest mass (m_n)	$1.675 \times 10^{-27}\,\mathrm{kg}$ ($940\,\mathrm{MeV/c^2}$)
W^+ rest mass (m_W)	$80.4\,\mathrm{GeV/c^2}$
Planck's constant (h)	$6.63 \times 10^{-34}\,\mathrm{J \cdot s}$
Speed of light in vacuum (c)	$3.00 \times 10^{8}\,\mathrm{m/s}$
Boltzmann's constant (k_B)	$1.38 \times 10^{-23}\,\mathrm{J/K}$
Gravitational constant (G)	$6.67 \times 10^{-11}\mathrm{N \cdot m^2/kg^2}$
Permeability of free space (μ_0)	$4\pi \times 10^{-7}\,\mathrm{H/m}$
Permittivity of free space (ϵ_0)	$8.85 \times 10^{-12}\,\mathrm{F/m}$
Mass of Earth (M_{Earth})	$5.98 \times 10^{24}\,\mathrm{kg}$
Mass of Moon (M_{Moon})	$7.35 \times 10^{22}\,\mathrm{kg}$
Radius of Earth (R_{Earth})	$6.38 \times 10^{6}\,\mathrm{m}$
Radius of Moon (M_{Moon})	$1.74 \times 10^{6}\,\mathrm{m}$
Radius of Sun (R_{Sun})	$6.96 \times 10^{8}\,\mathrm{m}$
Earth - Sun distance (R_{ES})	$1.50 \times 10^{11}\,\mathrm{m}$
Density of iron at low temperature (ρ_{Fe})	$7.88 \times 10^{3}\,\mathrm{kg/m^3}$
Classical electron radius (r_0)	$2.82 \times 10^{-15}\,\mathrm{m}$
Gravitational acceleration on Earth (g)	$9.8\,\mathrm{m/s^2}$
Atomic mass unit	$1.66 \times 10^{-27}\,\mathrm{kg}$
Specific heat of oxygen (c_V)	$21.1\,\mathrm{J/mole \cdot K}$
Specific heat of oxygen (c_P)	$29.4\,\mathrm{J/mole \cdot K}$

Moments of Inertia

For a hoop of mass M and radius R, about its symmetry axis: MR^2.

For a disk of mass M and radius R, about its symmetry axis: $(1/2)\,MR^2$.

For a solid sphere of mass M and radius R, about any symmetry axis: $(2/5)\,MR^2$.

For a spherical shell of mass M and radius R, about any symmetry axis: $(2/3)\,MR^2$.

Spherical harmonics

The spherical harmonics Y_{lm} have the normalization property

$$\int_{-1}^{1} d\cos\theta \int_{0}^{2\pi} d\phi \; Y^{\star}_{lm}(\theta,\phi)\, Y_{l'm'}(\theta,\phi) = \delta_{ll'}\, \delta_{mm'}.$$

The first few are

$$\begin{aligned}
Y_{00} &= \frac{1}{\sqrt{4\pi}} \\
Y_{11} &= -\sqrt{\frac{3}{8\pi}}\, \sin\theta\, e^{i\phi} \\
Y_{10} &= \sqrt{\frac{3}{4\pi}}\, \cos\theta \\
Y_{22} &= \sqrt{\frac{15}{32\pi}}\, \sin^2\theta\, e^{2i\phi} \\
Y_{21} &= -\sqrt{\frac{15}{8\pi}}\, \sin\theta\, \cos\theta\, e^{i\phi} \\
Y_{20} &= \sqrt{\frac{5}{16\pi}}\, [3\cos^2\theta - 1]
\end{aligned}$$

with $Y_{l,-m}(\theta,\phi) = -Y^{*}_{lm}(\theta,\phi)$.

Laplacian operator

Cartesian coordinates:

$$\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

Cylindrical coordinates:

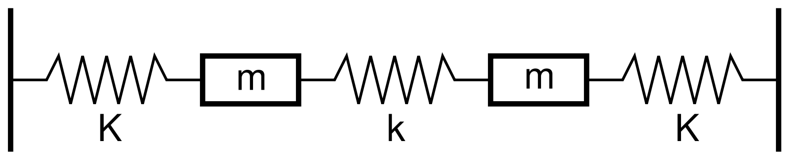
$$\nabla^2 t = \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial t}{\partial r}) + \frac{1}{r^2}\frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}$$

Spherical coordinates:

$$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial t}{\partial r}) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial t}{\partial\theta}) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$

Problem 1

Consider a system of two identical masses m connected by ideal, massless springs as shown. The middle spring constant is k, while the outer springs are identical with constant K, where $K \neq k$. The outer springs are attached to rigid walls. We consider only the horizontal, one-dimensional motion of the system.

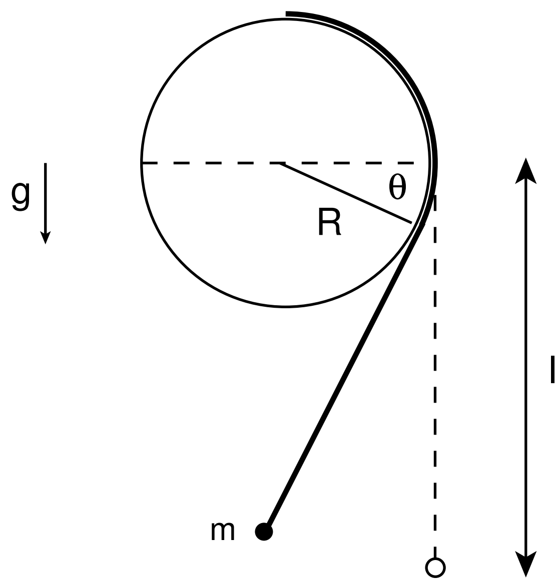


a) Write down the equations of motion for each mass.

b) Determine the normal modes of motion and the frequency of each mode.

Problem 2

A mass is suspended in a gravitational field by a string wrapped around a cylinder of radius R as shown. The string is attached at the top of the cylinder, and the mass is free to swing like a pendulum. Assume that the mass only swings in the vertical plane normal to the cylinder axis, the mass always remains below the cylinder, and the string always remains taut.



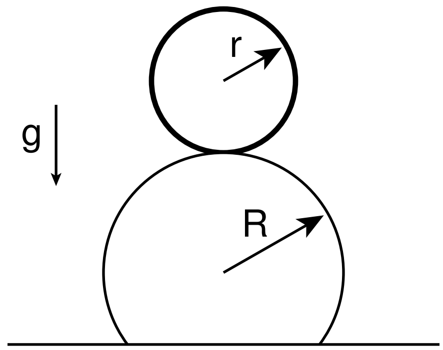
a) Write down the Lagrangian for this system taking the angle θ between the line from the center of the cylinder to the last point of string-cylinder contact and the horizontal as your independent dynamical variable. The length of the string *not* in contact with the cylinder when $\theta = 0$ is l.

b) Derive the equation of motion for θ.

c) Simplify this equation in the limit $R \ll l$. Do you recognize this result?

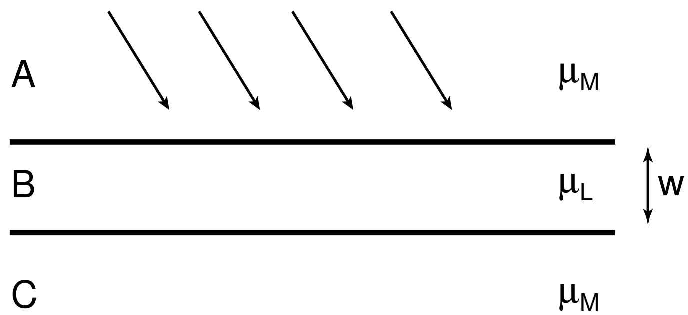
Problem 3

A uniform hoop of mass m and radius r rolls without slipping on a fixed cylinder of radius R as shown. The only external force is that of gravity. If the smaller hoop starts rolling from rest on top of the bigger cylinder, find the point at which the hoop falls off the cylinder.



Problem 4

A layer of material with a magnetic permeability of $\mu_L = 2$ is embedded in a medium of magnetic permeability of $\mu_M = 4 \times 10^3$. The layer has thickness w and in the other two dimensions its length approaches infinity. Both materials are uniform, isotropic, and non-conducting. We label the three regions as A, B, and C (see figure below). In region A, there is a magnetic field $\vec{H}$ with magnitude H_A at an angle $\alpha_A = \pi/6$ with respect to the normal to the interface.



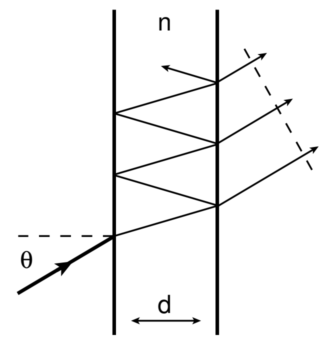
a) State the boundary conditions at the interface for the parallel and perpendicular components of the magnetic field.

b) In regions B and C, evaluate the magnitude of $\vec{H}$ in terms of H_A and the angle of the field with respect to the normal to the interface.

Problem 5

A grounded, conducting sphere of radius a is placed with its center at the origin. A charge Q is placed on the z-axis at $z = b$, where $b > a$. Find the force on the charge Q.

Problem 6



An electromagnetic plane wave with field amplitude E_0 and angular frequency ω is incident on a glass slab with thickness d and index of refraction n. As shown in the ray diagram, the wave goes through multiple reflections inside the slab. The initial angle of incidence is θ. The amplitude transmission coefficients (i.e. ratio of the transmitted and incident electric field amplitudes) from air to glass and from glass to air are t and t' respectively. The amplitude reflection coefficient (i.e ratio of the reflected and incident electric field amplitudes) in the glass is r. Assume that all the coefficients are real and positive.

a) Determine the phase difference at the transmitted wavefront between adjacent rays.

b) Write down the amplitude of each transmitted ray.

c) Determine the total intensity of the transmitted wave.

Note that: $r^2 + tt' = 1$