Exam #: ________________________

Printed Name: ________________________

Signature: ________________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Ph.D. Qualifying Examination
and
Master's Final Examination, PART II

Tuesday, 30 September 2008, 9:00 a.m. to 1:00 p.m.

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are eight equally weighted questions, each beginning on a new page. Read all eight questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic, and will be provided. **Personal calculators of any type are not allowed**. Paper dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries will not be allowed. No other papers or books may be used.**

When you have finished, come to the front of the room and hand your examination paper to the proctor; first put all problems in numerical order and staple them together.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand in your exam paper on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg (0.511 MeV/c^2)
Proton rest mass (m_p)	1.673×10^{-27} kg (938 MeV/c^2)
Neutron rest mass (m_n)	1.675×10^{-27} kg (940 MeV/c^2)
W^+ rest mass (m_W)	80.4 GeV/c^2
Planck's constant (h)	6.63×10^{-34} J$\cdot$ s
Speed of light in vacuum (c)	3.00×10^{8} m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Gravitational constant (G)	6.67×10^{-11}N$\cdot$ m^2/kg^2
Permeability of free space (μ_0)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space (ϵ_0)	8.85×10^{-12} F/m
Bohr magneton (μ_B)	9.27×10^{-24} J/T
Mass of Earth ($M_\oplus$)	5.98×10^{24} kg
Mass of Moon ($M_{\rm Moon}$)	7.35×10^{22} kg
Mass of Sun ($M_\odot$)	1.99×10^{30} kg
Radius of Earth ($R_\oplus$)	6.38×10^{6} m
Radius of Moon ($M_{\rm Moon}$)	1.74×10^{6} m
Radius of Sun ($R_\odot$)	6.96×10^{8} m
Earth - Sun distance ($R_{\oplus,\odot}$)	1.50×10^{11} m
Classical electron radius ($r_\circ$)	2.82×10^{-15} m
Gravitational acceleration on Earth (g)	9.8 m/s^2
Atomic mass unit	1.66×10^{-27} kg
One atmosphere (1 atm)	1.01×10^{5} N/m^2

Stirling's Approximation

$$\log N! \;\approx\; N \log N - N$$

Problem 1

The goal of this problem is to estimate the order of magnitude of the so-called degeneracy pressure that arises in nonrelativistic Fermi systems at temperature T = 0 due to the Pauli exclusion principle.

a) Calculate the quantum mechanical ground state energy of a particle of mass m confined to a cubic box of side L.

Now consider a system of N non-interacting, identical Fermi particles, each of mass m, confined to a volume V at temperature $T = 0$. Treat the effect of the Pauli exclusion principle by assuming that each particle is confined in its own cubic box, which does not overlap the boxes of the other particles, and which together fill the entire volume V. Do not worry about spin degeneracy factors; we're not looking for that accurate an answer.

b) What is total ground state energy of this entire system of N particles?

c) What is the pressure? Express your answer entirely in terms of fundamental constants, the mass m of the particle, and the number density $n = N/V$.

Problem 2

An electron of the hydrogen atom is in the (normalized) state described by the wave function (at $t = 0$):

$$\Psi(\mathbf{r}, 0) = \frac{1}{6}\left[5\psi_{100}(\mathbf{r}) - \sqrt{2}\psi_{200}(\mathbf{r}) + 3\psi_{211}(\mathbf{r})\right]. \tag{1}$$

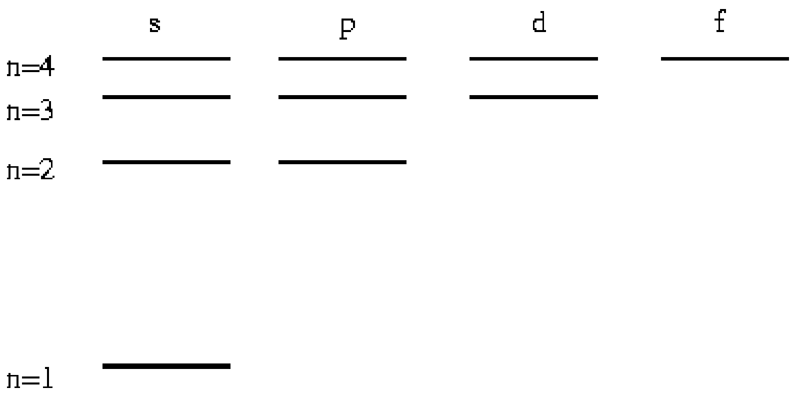
a) What is the expectation value of the energy? (Please express your answer in eV)

b) What are the expectation values of L^2 and L_z? (express your answer in units of $\hbar^2$ and $\hbar$, respectively).

c) What is the expectation value of L_z as a function of time?

Problem 3

The energies of the n=1,2,3, and 4 levels for an electron in atomic hydrogen are shown below.



a) Assume that an electron is initially in the $4p$ excited state of atomic hydrogen. Draw arrows on the diagram above to indicate all of the ways that electric dipole allowed transitions will take this initial state ultimately to the ground ($1s$) state (you do not need to indicate the values of the m quantum number).

b) For simplicity let us suppose that all of the matrix elements for the dipole-allowed transitions between the n=4 and n=3 levels are nearly equal (including the angular integrals). If the lifetime for the (400) state (which is due to the allowed $4s \rightarrow 3p$ transitions) is $\tau_\circ$, estimate the lifetimes for the (nlm)=(411) state, and for the (nlm)=(422) state. Be sure to explain your reasoning.

Problem 4

a) Find the energies of the 2 lowest lying quantum states for a particle of mass m confined to a 1-dimensional infinite square well with walls at $x = 0$ and $x = a$ (assume the potential energy is zero at the bottom of the square well).

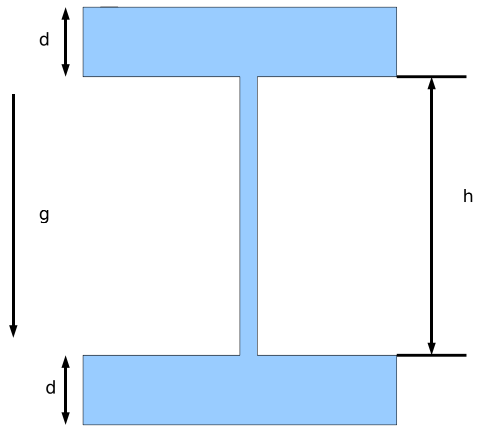
b) Suppose a weak repulsive δ-function potential of strength Γ (with units of energy$\times$length) is now placed at $x = a/2$; that is: $V(x)= \Gamma\delta(x\text{-}a/2)$. Using first order pertubation theory, calculate the new energies of the 2 lowest lying quantum states.

c) Now suppose the repulsive δ-function potential, still centered at $x = a/2$, is increased substantially in strength so that the energies of the two lowest energy eigenstates lie within 4% of each other. Without any detailed calculation, sketch the appearance of the two lowest eigenstates for such a situation.

d) The situation described in part (c) is relevant for the following problem: A particle is initially confined to one half of a container by a partially permeable partition. Estimate how long the particle will remain confined to that half of the container (and try to give a rough answer in terms of m, a, and fundamental constants).

Problem 5

Consider a classical mono-atomic gas of N non-interacting atoms, each of mass m, confined in a cell consisting of two identical large chambers, each of volume V, connected by a very thin vertical tube of length h, as shown in the figure. The tube is assumed to be so thin that a negligible number of particles will be in it at any time; it does, however, allow particles to move freely between the two chambers. The entire apparatus sits in a uniform vertical gravitational field of gravitational acceleration $\vec{g}$. The vertical thickness d of the two chambers is $\ll h$, and can be neglected for the purposes of this problem. The height h of the thin vertical tube can *not* be so neglected.



a) When the system is at equilibrium at a temperature $T > 0$, what is the average number of particles in the upper chamber?

b) What is the average energy of the system at this temperature, taking the gravitational potential energy in the lower chamber to be zero?

c) Roughly, plot the specific heat as a function of temperature T. Give a heuristic argument explaining the qualitative shape of your plot of the specific heat.

Problem 6

In this problem you will find the temperature of an adiabatic atmosphere versus height. Assume that the atmosphere is thin so that the atmosphere can be treated as planar and the gravitational acceleration can be held constant.

a) Find $\frac{dP}{dz}$, where P is the pressure and z is the height above the Earth's surface. Give your answer in terms of P, μ the molecular weight of the air, g the gravitational acceleration, R the gas constant and T the temperature. Assume an ideal gas.

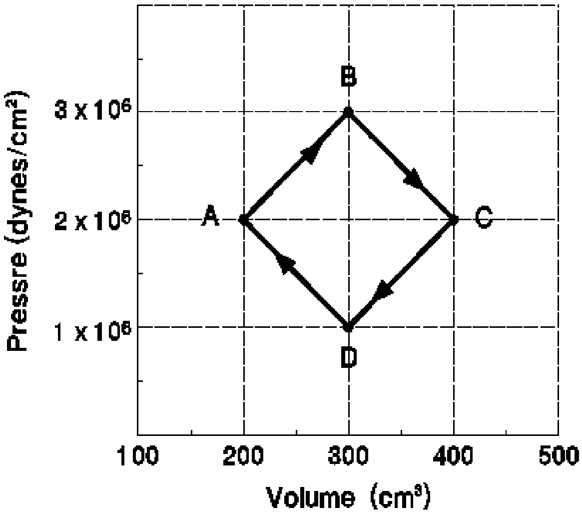
b) Find $\frac{dT}{dP}$ for an adiabatic expansion. Give your answer in terms of P, T, the gas constant R and $\gamma = c_p/c_v$.

c) Find the temperature as a function of height.

d) If the temperature is 20° C at the Earth's surface, what is it at 10 km? Assume air is made of N_2 ($\mu \simeq 28\ g$ and $R = 8.3\ \frac{J}{K\ mole}$).

Problem 7

Suppose that one mole of a monatomic ideal gas is employed in a heat engine which follows the cyclic quasi-static process represented in the P vs. V diagram shown. The highest temperature of the system occurs at point B (T_H) and the lowest temperature at point D (T_L).



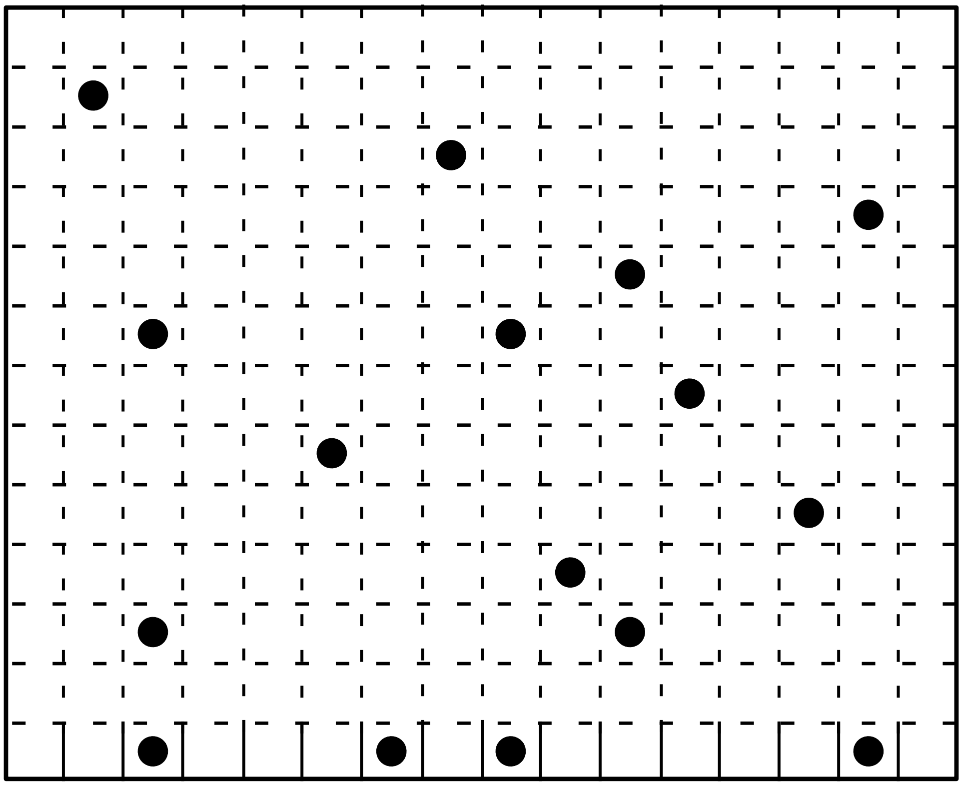
a) What is the work performed by this engine in one cycle?

b) Heat is absorbed in this cycle in going from A to C and is expelled going from C to A. Determine how much heat is absorbed in going from A to C.

c) What is the efficiency of this engine?

Problem 8

Physical adsorption (physisorption) is the process whereby gas molecules become attached to a surface by van der Waals forces. If this surface consists of the walls of a closed container, then we can study the process by considering the equilibrium between molecules in the gas phase and those on the walls. We can consider the walls as divided into A possible "adsorption sites." These are cells, each of which can accomodate a single molecule. Similarly we imagine that there are G possible gas-phase cells. There are A molecules in the container, enough to occupy all the adsorption sites under appropriate conditions. Suppose that there are N molecules on the surface of the container. When a molecule is physisorbed, it has energy -ε and when it in the gas phase, it has energy 0. Here $\varepsilon > 0$.



a) Find the entropy S as a function of N. Consider a system where $G \gg A \gg N \gg 1$. You may use Stirling's approximation in your calculation.

b) Find $(\partial S/\partial N)_{G,A}$ and $(\partial N/\partial E)_{G,A}$, where E is the total energy of the system.

c) Show that the number of physiabsorbed molecules N and the equilibrium temperature T are related as

$$\frac{(A-N)^2}{N(G-A+N)} = e^{-\varepsilon/kT}. \qquad (2)$$

d) Find the limit of N/A as $T \to \infty$ and comment.