Exam #: ______________________

Printed Name: ______________________

Signature: ______________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Ph.D. Qualifying Examination
and
Master's Final Examination, PART II

Wednesday, September 12, 2007, 1:00 p.m. to 5:00 p.m.

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are eight equally weighted questions, each beginning on a new page. Read all eight questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic, and will be provided. **Personal calculators of any type are not allowed**. Paper dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries will not be allowed. No other papers or books may be used.**

When you have finished, come to the front of the room and hand your examination paper to the proctor; first put all problems in numerical order and staple them together.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand your exam paper in on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg (0.511 MeV/c^2)
Proton rest mass (m_p)	1.673×10^{-27} kg (938 MeV/c^2)
Neutron rest mass (m_n)	1.675×10^{-27} kg (940 MeV/c^2)
W^+ rest mass (m_W)	80.4 GeV/c^2
Planck's constant (h)	6.63×10^{-34} J$\cdot$ s
Speed of light in vacuum (c)	3.00×10^8 m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Gravitational constant (G)	6.67×10^{-11}N$\cdot$ m^2/kg^2
Permeability of free space (μ_0)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space (ϵ_0)	8.85×10^{-12} F/m
Bohr magneton (μ_B)	9.27×10^{-24} J/T
Mass of Earth (M_{Earth})	5.98×10^{24} kg
Mass of Moon (M_{Moon})	7.35×10^{22} kg
Mass of Sun (M_{Sun})	1.99×10^{30} kg
Radius of Earth (R_{Earth})	6.38×10^6 m
Radius of Moon (M_{Moon})	1.74×10^6 m
Radius of Sun (R_{Sun})	6.96×10^8 m
Earth - Sun distance (R_{ES})	1.50×10^{11} m
Classical electron radius (r_0)	2.82×10^{-15} m
Gravitational acceleration on Earth (g)	9.8 m/s^2
Atomic mass unit	1.66×10^{-27} kg
One atmosphere (1 atm)	1.01×10^5 N/m^2

Problem 1

Consider a simple harmonic oscillator.

a) Write the Hamiltonian in terms of the momentum operator $\hat{p}$, the mass m, the angular frequency ω and the displacement operator $\hat{x}$.

b) Define the annihilation and creation operators, $\hat{a}$ and $\hat{a}^\dagger$, respectively:

$$\hat{a} = \sqrt{\frac{m\omega}{2\hbar}}(\hat{x} + \frac{i}{m\omega}\hat{p})$$

Express the Hamiltonian in terms of $\hat{a}$, $\hat{a}^\dagger$, ω and constants.

c) Define the number operator $\hat{N} \equiv \hat{a}^\dagger \hat{a}$ and let $|n\rangle$ be the eigenfunction of $\hat{N}$ with eigenvalue n. Show that $|n\rangle$ is an energy eigenstate, and obtain the energy spectrum.

Problem 2

A neutral, spin-$\frac{1}{2}$ particle with magnetic moment μ is at rest in an oscillating magnetic field:

$$\vec{B} = B_0 \hat{z} \cos(\omega t)$$

where B_0 and ω are constants.

a) Construct the Hamiltonian matrix for this system.

b) At $t = 0$ the electron is in the spin-up state with respect to the x-axis: $\chi(0) = \chi_+^{(x)}$. Determine $\chi(t)$ at any subsequent time by solving the time-dependent Schrödinger equation.

c) What is the probability that a measurement of S_x at time t yields $-\hbar/2$?

d) What is the minimum field B_0 required to guarantee that at some time a complete flip in S_x occurs?

Problem 3

A photon of energy E_0 scatters off an electron, initially at rest. Find the energy E of the outgoing photon as a function of the photon scattering angle θ.

Problem 4

The proton appears to be stable, but some theories predict it will decay. In this problem you are asked to obtain a lower limit on the proton lifetime, τ_p.

a) Obtain an expression for the number of protons in your body which will have decayed during your lifetime. Use the fact that the human lifetime, τ_H, is much smaller then the proton lifetime. Also assume the following: (1) Humans are made mostly of water (2) A human weighs 100 kg and lives for $\tau_H = 75$ years.

b) The human body has an upper limit on the amount of ionizing radiation it can withstand, with the lethal lifetime dose being 1 Joule/kg. Assuming the entire decaying proton's rest mass is converted into ionizing radiation which is deposited in the body, obtain a lower limit for the proton lifetime.

Problem 5

A solid is composed of a mole of non-interacting atoms, each with a magnetic moment of one Bohr magneton μ_B. An external magnetic field is applied, and the spins align either parallel or anti-parallel to the field.

a) Write down the partition function for this system and deduce the Helmholtz free energy and entropy, considering only spin degrees of freedom.

b) Assuming the system is maintained at a temperature of $4\,\mathrm{K}$, and the applied field is increased quasi-statically from $1\,\mathrm{T}$ to $10\,\mathrm{T}$, how much heat is transferred from the system to the thermal reservoir?

c) If the system is now thermally isolated and the field is decreased from $10\,\mathrm{T}$ to $1\,\mathrm{T}$, what is the final temperature of the system?

Problem 6

Consider a system that contains three non-interacting particles. The single-particle energy states that a particle can occupy are at energies $\varepsilon = 0$, ε_0, $2\varepsilon_0$, $3\varepsilon_0$, etc. The particles are excited by a total energy equal to $3\,\varepsilon_0$. This energy can be divided among three particles, shared by two particles, or it can all be given to one particle.

a) Using Maxwell-Boltzmann statistics (for distinguishable, classical particles) derive the probability that two of these three particles will be in the $\varepsilon = 0$ energy level.

b) What is the same probability using Bose-Einstein statistics?

c) What is the same probability using Fermi-Dirac statistics?

Problem 7

An air conditioning unit cools the interior of a building by extracting heat from the air inside the building at $20\,^\circ\mathrm{C}$ and exhausting it to the outside air at $40\,^\circ\mathrm{C}$. Both the air inside and outside the building can be treated as heat reservoirs, and you may assume that the device operates quasi-statically. Calculate the maximum rate at which heat can be extracted from the interior if the air conditioner is driven by a $1\,\mathrm{kW}$ motor.

Problem 8

The differential work required to magnetize a substance is $\delta W = H\,\delta M$, where H is the applied magnetic field and M is the magnetization.

a) Derive expressions for $(\partial S/\partial H)_{T,P}$ and $(\partial V/\partial H)_{T,P}$ that could be evaluated if $M(T, P, H)$ were known. Hint: consider the quantity $X = U - TS - HM + PV$.

b) What inference can be drawn from the third law of thermodynamics about $(\partial M/\partial T)_{H,P}$?

c) For an ideal paramagnet at low field and not too low temperature, the magnetization is observed to follow the law $M = \gamma H/T$, where the constant γ is called the Curie constant. If this relation held for all H less than some field strength and all T less than some fixed temperature, would it be consistent with the third law of thermodynamics? Why or why not?