Exam #: ______________________

Printed Name: ______________________

Signature: ______________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Ph.D. Qualifying Examination, PART II

Thursday, September 14, 2006, 1:00 p.m. to 5:00 p.m.

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are six equally weighted questions, each beginning on a new page. Read all six questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic, and will be provided. **Personal calculators of any type are not allowed**. Paper dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries will not be allowed. No other papers or books may be used.**

When you have finished, come to the front of the room and hand your examination paper to the proctor; first put all problems in numerical order and staple them together.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand your exam paper in on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	$1.60 \times 10^{-19}\,\mathrm{C}$
Electron rest mass (m_e)	$9.11 \times 10^{-31}\,\mathrm{kg}$ ($0.511\,\mathrm{MeV/c^2}$)
Proton rest mass (m_p)	$1.673 \times 10^{-27}\,\mathrm{kg}$ ($938\,\mathrm{MeV/c^2}$)
Neutron rest mass (m_n)	$1.675 \times 10^{-27}\,\mathrm{kg}$ ($940\,\mathrm{MeV/c^2}$)
W^+ rest mass (m_W)	$80.4\,\mathrm{GeV/c^2}$
Planck's constant (h)	$6.63 \times 10^{-34}\,\mathrm{J \cdot s}$
Speed of light in vacuum (c)	$3.00 \times 10^{8}\,\mathrm{m/s}$
Boltzmann's constant (k_B)	$1.38 \times 10^{-23}\,\mathrm{J/K}$
Gravitational constant (G)	$6.67 \times 10^{-11}\mathrm{N \cdot m^2/kg^2}$
Permeability of free space (μ_0)	$4\pi \times 10^{-7}\,\mathrm{H/m}$
Permittivity of free space (ϵ_0)	$8.85 \times 10^{-12}\,\mathrm{F/m}$
Mass of Earth (M_{Earth})	$5.98 \times 10^{24}\,\mathrm{kg}$
Mass of Moon (M_{Moon})	$7.35 \times 10^{22}\,\mathrm{kg}$
Radius of Earth (R_{Earth})	$6.38 \times 10^{6}\,\mathrm{m}$
Radius of Moon (M_{Moon})	$1.74 \times 10^{6}\,\mathrm{m}$
Radius of Sun (R_{Sun})	$6.96 \times 10^{8}\,\mathrm{m}$
Earth - Sun distance (R_{ES})	$1.50 \times 10^{11}\,\mathrm{m}$
Density of iron at low temperature (ρ_{Fe})	$7.88 \times 10^{3}\,\mathrm{kg/m^3}$
Classical electron radius (r_0)	$2.82 \times 10^{-15}\,\mathrm{m}$
Gravitational acceleration on Earth (g)	$9.8\,\mathrm{m/s^2}$
Atomic mass unit	$1.66 \times 10^{-27}\,\mathrm{kg}$
Specific heat of oxygen (c_V)	$21.1\,\mathrm{J/mole \cdot K}$
Specific heat of oxygen (c_P)	$29.4\,\mathrm{J/mole \cdot K}$

Spherical harmonics

The spherical harmonics Y_{lm} have the normalization property

$$\int_{-1}^{1} d\cos\theta \int_{0}^{2\pi} d\phi \; Y_{lm}^{\star}(\theta,\phi)\, Y_{l'm'}(\theta,\phi) = \delta_{ll'}\,\delta_{mm'}.$$

The first few are

$$\begin{aligned}
Y_{00} &= \frac{1}{\sqrt{4\pi}} \\
Y_{11} &= -\sqrt{\frac{3}{8\pi}}\,\sin\theta\, e^{i\phi} \\
Y_{10} &= \sqrt{\frac{3}{4\pi}}\,\cos\theta \\
Y_{22} &= \sqrt{\frac{15}{32\pi}}\,\sin^2\theta\, e^{2i\phi} \\
Y_{21} &= -\sqrt{\frac{15}{8\pi}}\,\sin\theta\,\cos\theta\, e^{i\phi} \\
Y_{20} &= \sqrt{\frac{5}{16\pi}}\,[3\cos^2\theta - 1]
\end{aligned}$$

with $Y_{l,-m}(\theta,\phi) = -Y_{lm}^{*}(\theta,\phi)$.

Laplacian operator

Cartesian coordinates:

$$\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

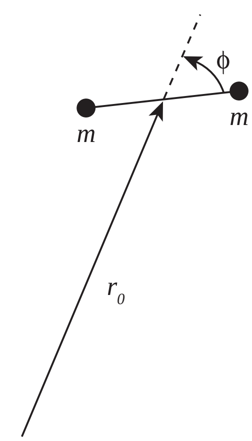
Cylindrical coordinates:

$$\nabla^2 t = \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial t}{\partial r}) + \frac{1}{r^2}\frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}$$

Spherical coordinates:

$$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial t}{\partial r}) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\frac{\partial t}{\partial \theta}) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial \phi^2}$$

Problem 1



A satellite moves in a circular orbit about the earth. The satellite consists of two equal masses, m. The masses are joined by a rigid rod of length ℓ and of negligible mass. Consider the masses to be point particles. The distance between the center of the earth and the center of mass of the satellite is r_0. Any motion of the masses is in the same plane as the satellite's orbit. The orientation of the satellite is described by the angle ϕ, as shown. Assume $\ell \ll r_0$.

a) Show that the potential energy can be expressed as

$$V(\phi) = -\frac{2GMm}{r_0}\left(1 + \frac{3\ell^2 \cos^2\phi}{8r_0^2}\right)$$

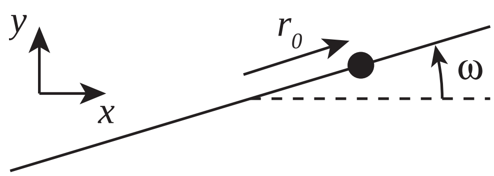
where M is the mass of the earth, assuming $\ell \ll r_0$.

b) Determine which values of ϕ correspond to positions of stable or unstable mechanical equilibrium.

c) Find the frequency of small oscillations about the stable equilibrium position(s) of ϕ.

Problem 2

A bead is free to move without friction along a rod rotating in the x-y plane about the z axis as shown in the figure below. The bead is initially at rest and located a distance r_0 from the center of rotation of the rod. The rod begins rotating with angular frequency ω at time $t = 0$.



a) Obtain the trajectory of the bead as a function of time.

b) Find the total energy of the bead.

c) As a function of time, find the work done per unit time on the bead.

Problem 3

Consider a single particle of charge e and mass m moving in a homogenous magnetic field $\vec{B}$, and *in addition* assume that a conservative, position-dependent force is present:

$$\vec{F}(\vec{r}) = -\vec{\nabla}V(\vec{r}).$$

The magnetic field is

$$\vec{B} = B\hat{z} = B\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

where $\hat{z}$ is the unit vector along the z axis and B is a constant. The potential energy is

$$V(\vec{r}) = V(x, y, z) = \frac{1}{2}m\omega_0^2 y^2,$$

i.e., it is independent of the coordinates x and z of the particle. Here ω_0 is a constant.

a) Show that the vector potential

$$\vec{A} \equiv -\frac{1}{2}\vec{r} \times \vec{B} = \frac{1}{2}B\begin{pmatrix} -y \\ +x \\ 0 \end{pmatrix}$$

gives rise to the above magnetic field $\vec{B}$.

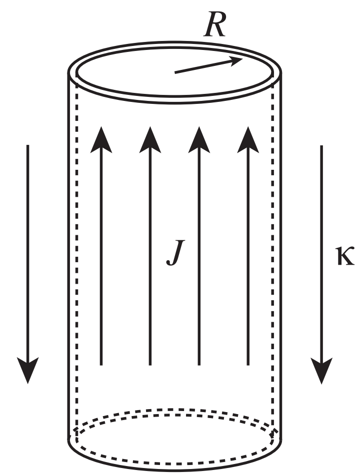
b) Write down the explicit form of the Largrangian $\mathcal{L}$ using the Cartesian coordinates r_i and their velocities $\dot{r}_i$ with the vector potential in a).

c) Find a gauge transformation that leads to a different Lagrangian $\mathcal{L}'$ in which x and y are cyclic variables.

d) For $\mathcal{L}'$ from c), write Lagrange's equations of motion for all three components of $\vec{r}$. Calculate the momenta conjugate to x, y, and z, and identify which of them are conserved.

Problem 4

A long cylindrical conductor of radius R carrying a current I in the $+\hat{z}$ direction has its current density J uniformly distributed over its circular cross section ($I = \pi R^2 J$). Suppose that the cylinder is coated with an insulating layer of thickness d and permeability of free space ($\mu = \mu_0$). A thin conducting film is then deposited over this insulating layer, allowing the current to return along the conducting surface layer in the $-\hat{z}$ direction ($I_{\text{surf}} = I = 2\pi(R+d)\kappa$). Assume $d \ll R$.



a) Find the magnetic field as a function of the radial distance s from the axis of the cylinder in terms of the current density J.

b) If the cylinder has a total length L, with $L \gg R$, find the total magnetic energy of this current configuration.

c) What is the self-inductance of this conducting circuit?

Problem 5

An electromagnetic wave with angular frequency ω is normally incident on a metal with conductivity σ, permittivity ϵ_0, and magnetic permeability μ_0. This problem can be analyzed by treating the metal as a dielectric with an effective permittivity $\epsilon_{\text{eff}} = \epsilon_0(1 + i\sigma/(\epsilon_0\omega))$.

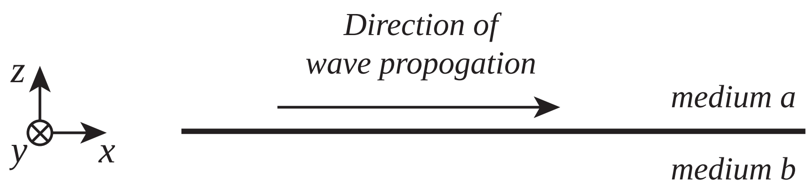
a) Assuming that the conductivity σ is real, find the wave number of the wave within the metal at low frequency and high frequency.

b) What is the dependence of the wave amplitude on distance from the surface and estimate the skin depth for a metal with $\sigma = 10^7(\Omega\,\mathrm{m})^{-1}$ at a wavelength of 500 nm.

c) We cannot actually assume that σ is independent of frequency. Above a certain frequency the conductivity will, in fact, become purely imaginary and we can write $\sigma \rightarrow i\sigma_\infty$. What is this frequency called? What is the phase velocity of the wave above this frequency? Comment on the value of the phase velocity.

Problem 6

Surface electromagnetic waves propagate along an interface between two media with their amplitudes decaying exponentially away from the interface. Consider the simple case of surface wave propagation at a planar interface between two semi-infinite isotropic media as shown:



a) Obtain the dispersion relation $k(\omega)$ for a transverse magnetic (TM) surface wave. Hint: write the electric field as

$$\vec{E} = (\hat{x}E_{ax} + \hat{z}E_{az})e^{(ikx - \alpha_a z - i\omega t)} \quad z > 0$$
$$\vec{E} = (\hat{x}E_{bx} + \hat{z}E_{bz})e^{(ikx - \alpha_b z - i\omega t)} \quad z < 0$$

where α_a and α_b are the field decay constants into the medium a and b, respectively.

b) What condition must hold for the dielectric constants of both media so that such a surface wave exists?

c) Show that a transverse electric (TE) wave cannot propagate as a surface wave.

d) A free-propagating electromagnetic wave is incident from medium a onto medium b at an angle. Is it possible to excite a surface-propagating wave under this condition? Explain.