Exam #: ______________________

Printed Name: ______________________

Signature: ______________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Ph.D. Qualifying Examination, PART II

Thurday, September 15, 2005, 1:00 p.m. to 5:00 p.m.

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are six equally weighted questions, each beginning on a new page. Read all six questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic. **Calculators with stored equations or text are not allowed**. Dictionaries may be used if they have been approved by the proctor before the examination begins. **No other papers or books may be used.**

When you have finished, come to the front of the room and hand your examination paper to the proctor; first put all problems in numerical order and staple them together.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand your exam paper in on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg (0.511 MeV/c^2)
Proton rest mass (m_p)	1.673×10^{-27} kg (938 MeV/c^2)
Neutron rest mass (m_n)	1.675×10^{-27} kg (940 MeV/c^2)
W^+ rest mass (m_W)	80.4 GeV/c^2
Planck's constant (h)	6.63×10^{-34} J$\cdot$ s
Speed of light in vacuum (c)	3.00×10^{8} m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Gravitational constant (G)	6.67×10^{-11}N$\cdot$ m^2/kg^2
Permeability of free space (μ_0)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space (ϵ_0)	8.85×10^{-12} F/m
Mass of Earth (M_{Earth})	5.98×10^{24} kg
Mass of Moon (M_{Moon})	7.35×10^{22} kg
Radius of Earth (R_{Earth})	6.38×10^{6} m
Radius of Moon (M_{Moon})	1.74×10^{6} m
Radius of Sun (R_{Sun})	6.96×10^{8} m
Earth - Sun distance (R_{ES})	1.50×10^{11} m
Density of iron at low temperature (ρ_{Fe})	7.88×10^{3} kg/m^3
Classical electron radius (r_0)	2.82×10^{-15} m
Gravitational acceleration on Earth (g)	9.8 m/s^2
Atomic mass unit	1.66×10^{-27} kg
Specific heat of oxygen (c_V)	21.1 J/mole$\cdot$ K
Specific heat of oxygen (c_P)	29.4 J/mole$\cdot$ K

Laplacian operator

Cartesian coordinates:

$$\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

Cylindrical coordinates:

$$\nabla^2 t = \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial t}{\partial r}) + \frac{1}{r^2}\frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}$$

Spherical coordinates:

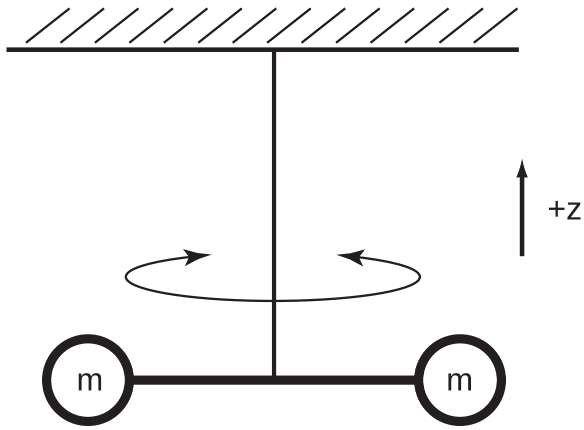
$$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial t}{\partial r}) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\frac{\partial t}{\partial \theta}) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 t}{\partial \phi^2}$$

Problem 1

Find the path of a particle of mass m in a central potential $U = \frac{1}{2}kr^2$ where r is the radial polar coordinate and k is a constant. Express your result as a function of initial position and velocity.

Problem 2

A torsion pendulum consists of two equal masses (m) attached to a rigid rod of length l that is suspended at its center by a long elastic fiber as shown. Torques (τ) around the fiber axis (z) cause the pendulum to twist in the xy plane by an angle θ. The fiber provides a restoring torque $-k\theta$.



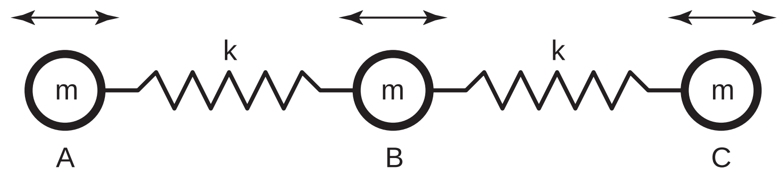
a) Find the moment of inertia I of the pendulum about the fiber axis assuming the masses m are much greater than the mass of the rod.

b) Assuming no friction, use Lagrangian techniques to write an equation of motion for the coordinate θ and an equation of motion for the momentum "p" that is canonical to θ.

c) Add a frictional torque term $-\gamma\dot{\theta}$ and solve the steady-state equation of motion in the presence of an oscillating applied torque $\tau(t) = \tau_0 \cos(\omega t)$. What is the resonance frequency of the torsional mode, assuming that $\gamma < \sqrt{kI}$?

Problem 3

Three identical masses m are connected by identical springs of spring constant k. The motion of the masses is confined to one dimension as shown below.



a) What are the natural frequencies and normal modes of the system?

b) If mass A is subjected to an external driving force $F(t) = F_0 \cos \omega t$ where $\omega = k/m$, find the steady-state motion of mass C.

Problem 4

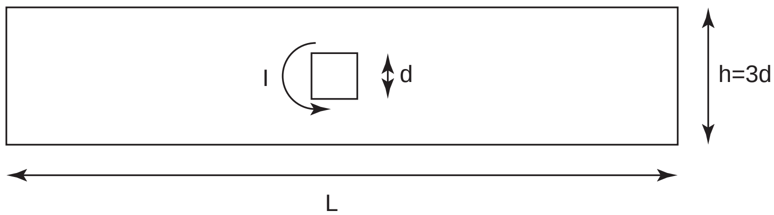
A laser beam impinges on the front of a planar mirror that reflects 99.9% and transmits 0.1% of the beam's intensity. You may model the laser beam as a monochromatic plane wave. Assume that the wave propagation is exactly normal to the mirror surface.

a) Suppose that you are given a second, identical mirror. Describe *in words* how you can arrange this mirror *behind* the first mirror such that the first mirror now *transmits* 100% of the beam intensity in steady state.

b) Show mathematically that your arrangement does, in fact, transmit all of the laser beam intensity. A sketch may help you to organize your solution. You may also find it useful to know the Stokes Relations: $r' = -r$ and $t't = 1 - r^2$. Here r and t are the reflection and transmission coefficients for the *field* (not intensity) for a beam incident on the front of the mirror, while r' and t' are the corresponding coefficients for a beam incident on the back of the same mirror. Ignore any thickness of the mirrors in your calculations.

Problem 5

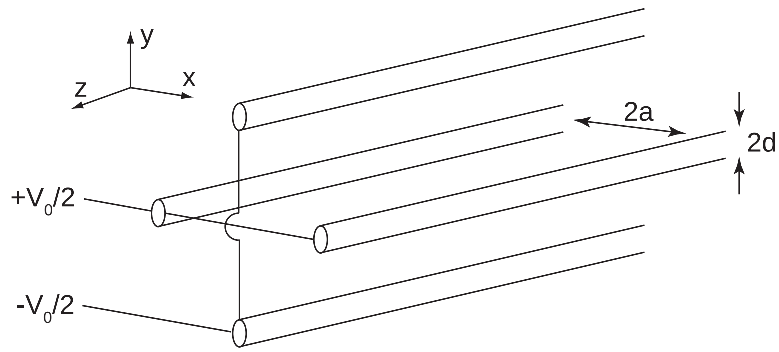
A small, square conducting loop of side d lies in the center of a large, rectangular conducting loop with sides of height $h = 3d$ and length L as shown. The resistances of the small and large loops are r and R respectively. The two loops lie in the same plane. The small loop has a counter-clockwise current I which is increasing at a constant rate k, that is $dI/dt = k$.



Find the current in the large loop (magnitude and direction). Assume that $L >> h$.

Problem 6

Four long cylindrical conductors are arranged parallel to one another on the corners of a square as shown. They are connected in pairs to a potential difference V_0. The conductors have radius d and are located at $(a, 0, z)$, $(-a, 0, z)$, $(0, a, z)$, and $(0, -a, z)$.



a) Sketch the equipotential lines in the xy plane.

b) Show that the potential a distance r from a *single* long cylindrical conductor with surface charge density ρ is given by

$$\phi(r) = -\frac{\rho d}{\epsilon_0} \ln \frac{r}{d} + V$$

where V is the potential of the conductor.

c) Show that close to the origin, the potential for the four conductor arrangement has the form

$$\phi(x, y) = A\frac{x^2 - y^2}{2a^2}, \quad r \geq d,$$

where A is a constant depending on V_0, a, and d. (You are not required to obtain an explicit expression for A.)

d) Can an arrangement like this be used to trap (confine near the origin) a positive ion moving in the xy plane? Why or why not?