Exam #: ______________________

Printed Name: ______________________

Signature: ______________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Ph.D. Qualifying Examination, PART III

Wednesday, April 4, 2007, 1:00 p.m. to 5:00 p.m.

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are six equally weighted questions, each beginning on a new page. Read all six questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic, and will be provided. **Personal calculators of any type are not allowed**. Paper dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries will not be allowed. No other papers or books may be used.**

When you have finished, come to the front of the room and hand your examination paper to the proctor; first put all problems in numerical order and staple them together.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand your exam paper in on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg (0.511 MeV/c^2)
Proton rest mass (m_p)	1.673×10^{-27} kg (938 MeV/c^2)
Neutron rest mass (m_n)	1.675×10^{-27} kg (940 MeV/c^2)
W^+ rest mass (m_W)	80.4 GeV/c^2
Planck's constant (h)	6.63×10^{-34} J$\cdot$ s
Speed of light in vacuum (c)	3.00×10^8 m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Gravitational constant (G)	6.67×10^{-11}N$\cdot$ m^2/kg^2
Permeability of free space (μ_0)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space (ϵ_0)	8.85×10^{-12} F/m
Mass of Earth (M_{Earth})	5.98×10^{24} kg
Mass of Moon (M_{Moon})	7.35×10^{22} kg
Radius of Earth (R_{Earth})	6.38×10^6 m
Radius of Moon (M_{Moon})	1.74×10^6 m
Radius of Sun (R_{Sun})	6.96×10^8 m
Earth - Sun distance (R_{ES})	1.50×10^{11} m
Density of iron at low temperature (ρ_{Fe})	7.88×10^3 kg/m^3
Classical electron radius (r_0)	2.82×10^{-15} m
Gravitational acceleration on Earth (g)	9.8 m/s^2
Atomic mass unit	1.66×10^{-27} kg
Specific heat of oxygen (c_V)	21.1 J/mole$\cdot$ K
Specific heat of oxygen (c_P)	29.4 J/mole$\cdot$ K

Problem 1

The lowering operator a reduces the energy of the harmonic oscillator stationary states:

$$a|n\rangle = \sqrt{n}|n-1\rangle$$

We define a *coherent state* $|\alpha\rangle$ as an eigenstate of the lowering operator:

$$a|\alpha\rangle = \alpha|\alpha\rangle$$

where the eigenvalue α may be any complex number. The coherent state may be expanded in terms of the energy eigenstates $|n\rangle$:

$$|\alpha\rangle = \sum_n c_0 \frac{\alpha^n}{\sqrt{n!}}|n\rangle$$

where $c_0 = e^{(|\alpha|^2/2)}$ is a normalization constant.

a) Assume a harmonic oscillator at $t = 0$ is in a coherent state $|\alpha\rangle$. Obtain an expression for the state of the system at a later time $t > 0$.

b) Show that while evolving in time, the system remains an eigenstate of the lowering operator, with an eigenvalue given by $\alpha(t) = \alpha e^{-i\omega t}$.

c) Express the position (x) and momentum (p) operators in terms of a and $a^\dagger$, and calculate the following expectation values for the harmonic oscillator above, as function of time:

$$\langle x\rangle,\ \langle x^2\rangle,\ \langle p\rangle,\ \langle p^2\rangle.$$

d) Using the results above determine the product $\sigma_x\sigma_p$, where σ_x and σ_p are the standard deviations in x and p respectively. Show that σ_x and σ_p are independent of time, and that

$$\sigma_x\sigma_p = \frac{\hbar}{2}$$

Problem 2

Neutrinos are spin-1/2 particles that have been experimentally observed to oscillate from one type (known as *flavor*) into another.

Consider two neutrino flavor eigenstates, $|\nu_e\rangle$ and $|\nu_\mu\rangle$, which are related to the energy eigenstates $|\nu_1\rangle$ and $|\nu_2\rangle$ by the transformation

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix} .$$

The energies corresponding to the energy eigenstates are

$$E_{1,2} = \sqrt{p^2c^2 + m_{1,2}^2 c^4}$$

where p is the momentum of our energy eigenstate (i.e. it does not change from one state to another.)

For simplicity, assume the super-relativistic limit $p \gg mc$, so that

$$E_{1,2} \simeq pc + \frac{m_{1,2}^2\, c^3}{2p} .$$

a) Calculate the time evolution of a neutrino flavor eigenstate.

b) Show that neutrino oscillations imply they have mass.

Problem 3

A point mass m moving vertically under the gravitational acceleration g follows the one-dimensional Schrdinger equation with a potential $V(x) = -mgx$, where x is the height above a defined zero-level.

a) Show by direct substitution that the time-dependent Schrdinger equation is solved by a wave packet of the form

$$\psi(x,t) = \int_{-\infty}^{\infty} dk\, e^{ikx - i\hbar^2 k^3/(6m^2 g)} A(\frac{\hbar k}{mg} - t),$$

where $A(z)$ is a function of one real variable. Hint: The form of $A(z)$ is arbitrary, we only require that it should be continuously differentiable and decay to zero at $z = \pm\infty$.

b) Write an integral expression for the expectation value of the momentum p_x in terms of the function A.

c) Now choose the particular form

$$A(\frac{\hbar k}{mg} - t) = \exp\{-b^2(\frac{\hbar k}{mg} - t + t_0)^2 + i\frac{mg^2}{6\hbar}(\frac{\hbar k}{mg} - t)^3\}$$

with real constants b and t_0. Show that this defines a wavepacket whose spatial width does not change over time, and for which $\langle p_x \rangle$ follows Newton's second law for a mass m accelerated by g.

Problem 4

Consider a model consisting of N two-level systems, each with a ground state and an excited state at energy ε where $\varepsilon > 0$. When the total energy of the system is U relative to its ground state:

a) Calculate the entropy as a function of U.

b) Calculate the temperature as a function of U and show that it can be negative.

c) When a system with negative temperature comes into contact with a system of positive temperature, what is the direction of heat flow? Why?

Problem 5

Consider a system of N identical, classical, non-interacting particles in one dimension confined by a quartic potential. The Hamiltonian of the system is

$$H(p_1, ..., p_N, x_1, ..., x_N) = \sum_{i=1}^{N} \left(\frac{p_i^2}{2m} + \alpha \left(\frac{x_i}{L} \right)^4 \right)$$

where p_i are the momenta and x_i are the positions, the strength parameter $\alpha \geq 0$, and L is a length characterizing the range of the quartic potential. Assume the system is in equilibrium at temperature T, and that classical statistical mechanics is applicable.

a) Show that the classical partition function Q in the canonical ensemble can be written as the product

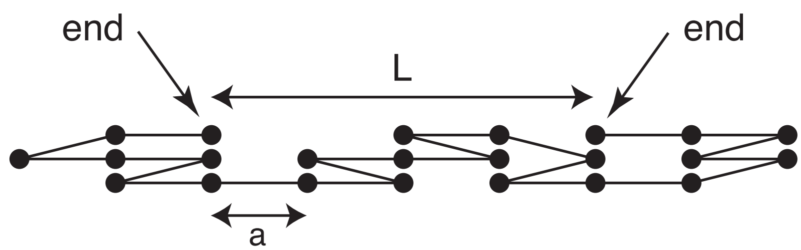
$$Q = Q_N^{\text{ideal}} f_N(L^4 kT/\alpha)$$

where Q_N^{ideal} is the partition function of an ideal gas with N particles and $f_N(L^4 kT/\alpha)$ is some function of the combination $L^4 kT/\alpha$.

b) Find an expression for the energy and heat capacity of this system as a function of T. Compare your answer to the classical equipartition theorem.

c) At time t the system is at temperature T_0 and the strength parameter has value α_0. The strength parameter is then varied adiabatically to a final value α. What is the temperature T of the system following this adiabatic process? That is, find the adiabatic exponent connecting T and α.

Problem 6



Consider a chain of $N \gg 1$ rigid segments of length a lying in a line. Each segment can be folded back on the previous segment or not, as shown above. Folded and unfolded links have the same energy. The chain is in thermal equilibrium at temperature T.

a) Find expressions for the free energy and entropy of this system.

b) Calculate the force F needed to hold the ends of this chain a distance L apart for all values of L in the range $a < L < Na$. (Hint: F and L for this one-dimensional system are analogous to P and V, respectively, for a three-dimensional gas).