Exam #: ________________________

Printed Name: ________________________

Signature: ________________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Ph.D. Qualifying Examination, PART III

Wednesday, April 5, 2006, 1:00 p.m. to 5:00 p.m.

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are six equally weighted questions, each beginning on a new page. Read all six questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

You will not need calculators for this exam. **Calculators or any other electronic devices are not allowed, including electronic dictionaries**. Paper dictionaries may be used if they have been approved by the proctor before the examination begins. **No other papers or books may be used.**

When you have finished, come to the front of the room and hand your examination paper to the proctor; first put all problems in numerical order and staple them together.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand your exam paper in on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg (0.511 MeV/c^2)
Proton rest mass (m_p)	1.673×10^{-27} kg (938 MeV/c^2)
Neutron rest mass (m_n)	1.675×10^{-27} kg (940 MeV/c^2)
W^+ rest mass (m_W)	80.4 GeV/c^2
Planck's constant (h)	6.63×10^{-34} J$\cdot$ s
Speed of light in vacuum (c)	3.00×10^8 m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Gravitational constant (G)	6.67×10^{-11}N$\cdot$ m^2/kg^2
Permeability of free space (μ_0)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space (ϵ_0)	8.85×10^{-12} F/m
Mass of Earth (M_{Earth})	5.98×10^{24} kg
Mass of Moon (M_{Moon})	7.35×10^{22} kg
Radius of Earth (R_{Earth})	6.38×10^6 m
Radius of Moon (M_{Moon})	1.74×10^6 m
Radius of Sun (R_{Sun})	6.96×10^8 m
Earth - Sun distance (R_{ES})	1.50×10^{11} m
Density of iron at low temperature (ρ_{Fe})	7.88×10^3 kg/m^3
Classical electron radius (r_0)	2.82×10^{-15} m
Gravitational acceleration on Earth (g)	9.8 m/s^2
Atomic mass unit	1.66×10^{-27} kg
Specific heat of oxygen (c_V)	21.1 J/mole$\cdot$ K
Specific heat of oxygen (c_P)	29.4 J/mole$\cdot$ K

Problem 1

Consider the following operators on a Hilbert space:

$$
\begin{aligned}
L_x &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \\
L_y &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \\
L_z &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.
\end{aligned}
$$

a) Consider the state with L_z eigenvalue $+1$. In this state, what are $\langle L_x \rangle$ and $\langle L_x^2 \rangle$?

b) Find the normalized eigenstates and eigenvalues of L_x in the L_z basis.

c) If the particle is in the $L_z = -1$ state and L_x is measured, what are the possible outcomes and their probabilities?

Problem 2

It is known that in the absence of external perturbations, a certain quantum system has only two degenerate eigenstates $|a\rangle$ and $|b\rangle$. Define the energy of each of these states to be zero. In the presence of a constant external field F, the new energy eigenstates are non-degenerate, with energies $\pm\delta$.

a) Initially the system is known to be in state $|a\rangle$, then at time $t = 0$ the external field F is suddenly turned on and maintained at a constant strength, causing the system to evolve in time. Solve the Schrödinger equation exactly for this time evolution.

b) After a time interval equal to T, during which the system evolves as described above, you make a measurement that determines which of the following two orthogonal superposition states the system is in:

$$\begin{aligned} |c\rangle &= \frac{1}{\sqrt{2}}(|a\rangle + |b\rangle) \\ |d\rangle &= \frac{1}{\sqrt{2}}(|a\rangle - |b\rangle). \end{aligned}$$

What are the probabilities of finding the system in these states?

Problem 3

a) Prove the three-dimensional virial theorem valid in Quantum Mechanics for stationary states:

$$2\langle T\rangle = \langle \vec{r}\cdot\nabla V\rangle,$$

where T and V are the kinetic and potential energies respectively of a particle and $\vec{r}$ is its position.

Hint: In classical (i.e. non-quantum) mechanics one usually starts by evaluating the time derivative of the product $\vec{r}\cdot\vec{p}$.

b) Apply this theorem to a three-dimensional isotropic harmonic oscillator that is in an eigenstate of energy and angular momentum $|n,l,m\rangle$ with energy $E=\hbar\omega(n+3/2)$ to express $\langle T\rangle$ in terms of n.

Problem 4

Consider a system of two identical particles and two discrete energy eigenstates. Each particle can possibly be in either state 1 with energy 0 or state 2 with energy $\varepsilon > 0$. Write down the (canonical) partition function for the system if:

a) the particles are distinguishable,

b) the particles are indistinguishable classical (Boltzmann) particles,

c) the particles are indistinguishable fermions,

d) the particles are indistinguishable bosons.

Problem 5

An aqueous solution at room temperature, T, contains a small concentration of magnetic atoms, each of which has a net spin of 1/2 and a magnetic moment of m.

The solution is placed in a an external magnetic field pointing in the z direction which varies in the z direction as $\vec{B} = B(z)\hat{z}$. Assume specifically that B has the value B_1 at the bottom of the solution where $z = z_1$ and a larger value B_2 at the top of the solution where $z = z_2$, and increases approximately linearly between z_1 and z_2. The particles do not interact with each other.

a) Let $n^+(z)dz$ denote the mean number of magnetic atoms whose spin points in the "up" or in the $+\hat{z}$ direction that are located between z and $z + dz$. What will be the fraction of spins pointing up at a height z, where $z_1 < z < z_2$?

b) What will be the average energy difference between a magnetic atom located at the bottom of the solution near z_1 compared to that at the top of the solution near z_2? (Ignore any effects due to gravity.) Make use of the fact that $mB \ll k_B T$ to simplify your answer.

c) Deduce the density ratio between the magnetic atoms near the top and near the bottom of the solution in thermal equilibrium. It may be helpful to recall that, for a system of dilute non-interacting, non-magnetic particles, the chemical potential is given by $\mu = k_B T \ln(n/n_Q)$, where n is the density and n_Q (the so-called quantum concentration) is simply a constant for a given type of particle at fixed temperature.

Problem 6

The Van der Waals equation of state for one mole of a fluid is,

$$(V - b)(P + a/V^2) = RT,$$

where P, T, and $V > b$ are respectively the pressure, temperature, and volume of the fluid; b and a are positive constants specific to the fluid; and R is the gas constant.

a) On a P-V diagram, for $V > b$, sketch out isotherms for the cases when RT is large compared to a/b, and when RT is small compared to a/b.

b) Show that there are either one or three real values of $V > b$ for each value of T and P; and by considering isothermal compressibility, or otherwise, show that when there are three real solutions, a particular one of them is unphysical.

c) When the fluid is compressed isothermally at low temperature, what determines the volume at which the system leaves the isotherm in part a), and what path does it follow in the P-V plane?