Exam #: ______________________

Printed Name: ______________________

Signature: ______________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Ph.D. Qualifying Examination, PART III

Wednesday, March 30, 2005, 1:00 p.m. to 5:00 p.m.

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are six equally weighted questions, each beginning on a new page. Read all six questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic. **Calculators with stored equations or text are not allowed**. Dictionaries may be used if they have been approved by the proctor before the examination begins. **No other papers or books may be used.**

When you have finished, come to the front of the room and hand your examination paper to the proctor; first put all problems in numerical order and staple them together.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand your exam paper in on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	$1.60 \times 10^{-19}\,\mathrm{C}$
Electron rest mass (m_e)	$9.11 \times 10^{-31}\,\mathrm{kg}$ ($0.511\,\mathrm{MeV/c^2}$)
Proton rest mass (m_p)	$1.673 \times 10^{-27}\,\mathrm{kg}$ ($938\,\mathrm{MeV/c^2}$)
Neutron rest mass (m_n)	$1.675 \times 10^{-27}\,\mathrm{kg}$ ($940\,\mathrm{MeV/c^2}$)
W^+ rest mass (m_W)	$80.4\,\mathrm{GeV/c^2}$
Planck's constant (h)	$6.63 \times 10^{-34}\,\mathrm{J\cdot s}$
Speed of light in vacuum (c)	$3.00 \times 10^{8}\,\mathrm{m/s}$
Boltzmann's constant (k_B)	$1.38 \times 10^{-23}\,\mathrm{J/K}$
Gravitational constant (G)	$6.67 \times 10^{-11}\mathrm{N\cdot m^2/kg^2}$
Permeability of free space (μ_0)	$4\pi \times 10^{-7}\,\mathrm{H/m}$
Permittivity of free space (ϵ_0)	$8.85 \times 10^{-12}\,\mathrm{F/m}$
Mass of Earth (M_{Earth})	$5.98 \times 10^{24}\,\mathrm{kg}$
Mass of Moon (M_{Moon})	$7.35 \times 10^{22}\,\mathrm{kg}$
Radius of Earth (R_{Earth})	$6.38 \times 10^{6}\,\mathrm{m}$
Radius of Moon (M_{Moon})	$1.74 \times 10^{6}\,\mathrm{m}$
Radius of Sun (R_{Sun})	$6.96 \times 10^{8}\,\mathrm{m}$
Earth - Sun distance (R_{ES})	$1.50 \times 10^{11}\,\mathrm{m}$
Density of iron at low temperature (ρ_{Fe})	$7.88 \times 10^{3}\,\mathrm{kg/m^3}$
Classical electron radius (r_0)	$2.82 \times 10^{-15}\,\mathrm{m}$
Gravitational acceleration on Earth (g)	$9.8\,\mathrm{m/s^2}$
Atomic mass unit	$1.66 \times 10^{-27}\,\mathrm{kg}$
Specific heat of oxygen (c_V)	$21.1\,\mathrm{J/mole\cdot K}$
Specific heat of oxygen (c_P)	$29.4\,\mathrm{J/mole\cdot K}$

Moments of Inertia

For a hoop of mass M and radius R, about its symmetry axis: MR^2.

For a disk of mass M and radius R, about its symmetry axis: $(1/2)\,MR^2$.

For a solid sphere of mass M and radius R, about any symmetry axis: $(2/5)\,MR^2$.

For a spherical shell of mass M and radius R, about any symmetry axis: $(2/3)\,MR^2$.

Spherical harmonics

The spherical harmonics Y_{lm} have the normalization property

$$\int_{-1}^{1} d\cos\theta \int_0^{2\pi} d\phi \; Y^{\star}_{lm}(\theta,\phi)\, Y_{l'm'}(\theta,\phi) = \delta_{ll'}\, \delta_{mm'}.$$

The first few are

$$\begin{aligned}
Y_{00} &= \frac{1}{\sqrt{4\pi}} \\
Y_{11} &= -\sqrt{\frac{3}{8\pi}}\, \sin\theta\, e^{i\phi} \\
Y_{10} &= \sqrt{\frac{3}{4\pi}}\, \cos\theta \\
Y_{22} &= \sqrt{\frac{15}{32\pi}}\, \sin^2\theta\, e^{2i\phi} \\
Y_{21} &= -\sqrt{\frac{15}{8\pi}}\, \sin\theta\, \cos\theta\, e^{i\phi} \\
Y_{20} &= \sqrt{\frac{5}{16\pi}}\, [3\cos^2\theta - 1]
\end{aligned}$$

with $Y_{l,-m}(\theta,\phi) = -Y^*_{lm}(\theta,\phi)$.

Laplacian operator

Cartesian coordinates:

$$\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

Cylindrical coordinates:

$$\nabla^2 t = \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial t}{\partial r}) + \frac{1}{r^2}\frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}$$

Spherical coordinates:

$$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial t}{\partial r}) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\frac{\partial t}{\partial \theta}) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial \phi^2}$$

Problem 1

To model an electron in a planar semiconductor quantum well, we assume that its motion is confined to the x-y plane. In this infinite two-dimensional space, consider the analogue of the hydrogen atom.

a) Write down the time-independent Schrödinger equation for the case of the two-dimensional Coulomb potential

$$V(\rho) = -\frac{k}{\rho}$$

where $k > 0$ and $\rho = \sqrt{x^2 + y^2}$ is the the radial polar coordinate.

b) Look for angular-momentum eigenfunctions that are solutions of this equation and give their ϕ dependence, where ϕ is the azimuthal angle in polar coordinates. What boundary condition(s) determine the ϕ dependent part of the wave function?

c) Write down the radial equation. What boundary condition(s) does the solution to this equation have to satisfy.

Problem 2

Consider the following Hamiltonian

$$H = A\mathbf{L}^2 + BL_z$$

where $\mathbf{L}$ is the total angular momentum operator and L_z is its z component.

a) Write down the eigenvalues of this Hamiltonian in terms of the eigenvalues of $\mathbf{L}^2$ and L_z.

b) A perturbation of the form

$$H_1 = CL_x$$

is added to H. For an arbitrary unperturbed eigenvalue of H as obtained in a), calculate the *first-order* energy correction.

c) For the same unperturbed eigenstate, calculate the *second-order* correction to the energy.

Hint: For two angular-momentum eigenstates $|l, m\rangle$ and $|l', m'\rangle$,

$$\langle l', m'|L_x \pm iL_y|l, m\rangle = \hbar\sqrt{(l \mp m)\,(l \pm m + 1)}\,\delta_{l',l}\,\delta_{m',m\pm1}.$$

Problem 3

A one-dimensional harmonic oscillator of mass m and natural frequency ω is in its ground state. At $t = 0$, a uniform force F is applied in the x direction, and at a time $t = T$ the force is switched off.

a) Write down the expression for the perturbation potential for times $0 < t < T$.

b) Calculate the matrix element of the perturbation potential between ground state and first excited state.

d) Calculate (in first order perturbation theory) the probability of finding the particle in its first excited state after the perturbation has been turned off. For what value(s) of T is this probability a maximum?

Problem 4

Consider a random walk in one dimension with variable step size. For each step, the probability density for the displacement x from the current position is

$$p(x) = A\, e^{-|x|/a}$$

where x is any real number, a is a positive real number and A is a normalization constant.

a) What is the *mean* of the net displacement from the starting point after 5 independent steps?

b) What is the *variance* of the net displacement from the starting point after 5 independent steps?

Problem 5

Consider a system whose equilibrium state can be completely described by the variables P (pressure), V (volume) and T (temperature).

a) For this system, write an expression for the total differential change in internal energy, dU, as a function of differential changes in entropy (dS) and volume (dV).

b) The coefficient of thermal expansion is

$$\alpha \equiv \frac{1}{V} \left.\frac{\partial V}{\partial T}\right|_P .$$

Show that this coefficient also determines the rate at which the entropy changes with pressure according to

$$\left(\frac{\partial S}{\partial P}\right)_T = -\alpha V.$$

Hint: identify the preferred variables and transform to the appropriate thermodynamic potential.

c) Evaluate α for an ideal monatomic gas and state if the entropy decreases or increases with rising pressure at constant T.

Problem 6

A sample of an **ideal gas** is spun in a centrifuge generating a constant acceleration a parallel to the axis of a cylindrical sample holder of length H. The gas contains N_A molecules of mass m_A and N_B molecules of mass m_B in equilibrium at temperature T.

a) For the molecules of each mass, find the number density distribution $n(h)$ as a function of distance h along the cylindrical axis of the sample holder.

b) At what value of h should the sample holder be partitioned to maximize the separation of the masses (maximize the difference in the number of heavy and light molecules in the more dense partition).