Exam #: ________________________

Printed Name: ________________________

Signature: ________________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Ph.D. Qualifying Examination, PART III

Friday, September 15, 2006, 1:00 p.m. to 5:00 p.m.

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are six equally weighted questions, each beginning on a new page. Read all six questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic, and will be provided. **Personal calculators of any type are not allowed**. Paper dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries will not be allowed. No other papers or books may be used.**

When you have finished, come to the front of the room and hand your examination paper to the proctor; first put all problems in numerical order and staple them together.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand your exam paper in on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg (0.511 MeV/c^2)
Proton rest mass (m_p)	1.673×10^{-27} kg (938 MeV/c^2)
Neutron rest mass (m_n)	1.675×10^{-27} kg (940 MeV/c^2)
W^+ rest mass (m_W)	80.4 GeV/c^2
Planck's constant (h)	6.63×10^{-34} J$\cdot$ s
Speed of light in vacuum (c)	3.00×10^8 m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Gravitational constant (G)	6.67×10^{-11}N$\cdot$ m^2/kg^2
Permeability of free space (μ_0)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space (ϵ_0)	8.85×10^{-12} F/m
Mass of Earth (M_{Earth})	5.98×10^{24} kg
Mass of Moon (M_{Moon})	7.35×10^{22} kg
Radius of Earth (R_{Earth})	6.38×10^6 m
Radius of Moon (M_{Moon})	1.74×10^6 m
Radius of Sun (R_{Sun})	6.96×10^8 m
Earth - Sun distance (R_{ES})	1.50×10^{11} m
Density of iron at low temperature (ρ_{Fe})	7.88×10^3 kg/m^3
Classical electron radius (r_0)	2.82×10^{-15} m
Gravitational acceleration on Earth (g)	9.8 m/s^2
Atomic mass unit	1.66×10^{-27} kg
Specific heat of oxygen (c_V)	21.1 J/mole$\cdot$ K
Specific heat of oxygen (c_P)	29.4 J/mole$\cdot$ K

Problem 1

A plane wave of wave number k, representing a particle of mass m, is incident from the left on the potential

$$
\begin{aligned}
V(x) \quad &= 0, \qquad x < 0 \\
&= \tfrac{8}{9}\tfrac{(\hbar k)^2}{2m}, \quad 0 < x < a \\
&= \tfrac{5}{9}\tfrac{(\hbar k)^2}{2m}, \quad a < x
\end{aligned}
$$

Find the ratio of the transmitted flux to the incident flux as a function of ka.

Problem 2

Outside a metal surface, electrons are attracted by a force which is approximately that of an image charge. Inside, the metal electronic states are occupied and so the extra electron is excluded, which may be taken into account by including an effective potential inside the metal, or by a boundary condition on the wave functions at the surface of the metal.

a) Write down a time-independent Schrödinger equation for an electron near a flat metal surface, specifying either the potential inside the metal or the boundary condition at the surface.

b) Find the energy of the ground state of the external electron bound to the metal surface.

c) Assume that an electron is moving parallel to the surface with a wave number k. What is the wave function and energy of this electron?

Problem 3

Consider N identically prepared spin-1/2 particles, each in the eigenstate where the spin points in the $+\hat{x}$ direction. Suppose each of the spins is measured along an axis $\hat{u}$ that subtends an angle of $\pi/4$ with the x direction. Calculate the probability of obtaining the outcome $+u$ exactly n times out of N measurements.

Problem 4

Consider a molecule shaped like an equilateral triangle with a spin at each vertex. The interaction energy in the presence of an external magnetic field is given by

$$E = J \sum_{\langle ij \rangle} S_i S_j - \mu H \sum_i S_i$$

where the spins $S_i = \pm 1$, $J > 0$, μ is the same magnetic moment for all three spins, and the first sum is taken over all pairs of spins. Ignore translational, rotational, and vibrational degrees of freedom.

a) Find the partition function.

b) Find an expression for the average total spin of the molecule $\langle S \rangle$.

c) Comment on the value of the average spin when $k_B T \ll |J|$ as a function of magnetic field.

d) Would your answers change if the sites of the spins at the vertices were not distinguishable? Justify your answer.

Problem 5

A DNA molecule consists of bases on two chains that behave something like a zipper with N links. Each link can be in one of two states: i) the link is closed with energy 0, or ii) the link is open with energy $\varepsilon > 0$. Assume that link r can be open only if all the links $1, 2, ..., r - 1$ are open, and also that if link r is closed, then all the links $r + 1, r + 2, ..., N$ must also be closed.

a) What is the free energy of this system at temperature T?

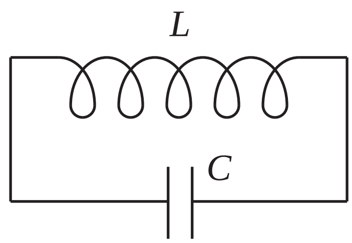
b) At low temperature, $k_B T \ll \varepsilon$, what is the mean number of open links?

c) At temperatures such that $k_B T \gg \varepsilon$ but $k_B T \ll N\varepsilon$, what is the mean number of open links?

d) At high temperature, $k_B T \gg N\varepsilon$, what is the mean number of open links?

Problem 6

Consider a series LC circuit in equilibrium with a heat reservoir at temperature T.



a) Find the average energy in the circuit.

b) Find the mean square noise voltage $\langle V^2 \rangle$ across either element. Examine $\langle V^2 \rangle$ in the high and low temperature limits.