Exam #: ____________________

Printed Name: ____________________

Signature: ____________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Ph.D. Qualifying Examination, PART III

Friday, September 16, 2005, 1:00 p.m. to 5:00 p.m.

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are six equally weighted questions, each beginning on a new page. Read all six questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic. **Calculators with stored equations or text are not allowed**. Dictionaries may be used if they have been approved by the proctor before the examination begins. **No other papers or books may be used.**

When you have finished, come to the front of the room and hand your examination paper to the proctor; first put all problems in numerical order and staple them together.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand your exam paper in on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	$1.60 \times 10^{-19}\,\mathrm{C}$
Electron rest mass (m_e)	$9.11 \times 10^{-31}\,\mathrm{kg}\ (0.511\,\mathrm{MeV/c^2})$
Proton rest mass (m_p)	$1.673 \times 10^{-27}\,\mathrm{kg}\ (938\,\mathrm{MeV/c^2})$
Neutron rest mass (m_n)	$1.675 \times 10^{-27}\,\mathrm{kg}\ (940\,\mathrm{MeV/c^2})$
W^+ rest mass (m_W)	$80.4\,\mathrm{GeV/c^2}$
Planck's constant (h)	$6.63 \times 10^{-34}\,\mathrm{J \cdot s}$
Speed of light in vacuum (c)	$3.00 \times 10^{8}\,\mathrm{m/s}$
Boltzmann's constant (k_B)	$1.38 \times 10^{-23}\,\mathrm{J/K}$
Gravitational constant (G)	$6.67 \times 10^{-11}\mathrm{N \cdot m^2/kg^2}$
Permeability of free space (μ_0)	$4\pi \times 10^{-7}\,\mathrm{H/m}$
Permittivity of free space (ϵ_0)	$8.85 \times 10^{-12}\,\mathrm{F/m}$
Mass of Earth (M_{Earth})	$5.98 \times 10^{24}\,\mathrm{kg}$
Mass of Moon (M_{Moon})	$7.35 \times 10^{22}\,\mathrm{kg}$
Radius of Earth (R_{Earth})	$6.38 \times 10^{6}\,\mathrm{m}$
Radius of Moon (M_{Moon})	$1.74 \times 10^{6}\,\mathrm{m}$
Radius of Sun (R_{Sun})	$6.96 \times 10^{8}\,\mathrm{m}$
Earth - Sun distance (R_{ES})	$1.50 \times 10^{11}\,\mathrm{m}$
Density of iron at low temperature (ρ_{Fe})	$7.88 \times 10^{3}\,\mathrm{kg/m^3}$
Classical electron radius (r_0)	$2.82 \times 10^{-15}\,\mathrm{m}$
Gravitational acceleration on Earth (g)	$9.8\,\mathrm{m/s^2}$
Atomic mass unit	$1.66 \times 10^{-27}\,\mathrm{kg}$
Specific heat of oxygen (c_V)	$21.1\,\mathrm{J/mole \cdot K}$
Specific heat of oxygen (c_P)	$29.4\,\mathrm{J/mole \cdot K}$

Useful Relations

$$\sin\alpha \sin\beta = \frac{1}{2}(\sin(\alpha+\beta) + \sin(\alpha-\beta))$$

$$\int x \sin bx = \frac{1}{b^2}\sin bx - \frac{x}{b}\cos bx$$

Binomial Formula

$$(a+b)^n = \sum_{k=0}^{n} \left(\begin{array}{c} n \\ k \end{array} \right) a^{n-k}\, b^k$$

Problem 1

Beryllium in its electrically neutral state has 4 electrons. Its ground state in orbital notation is $(1s)^2(2s)^2$ Its lowest excited states have the orbital configuration $(1s)^2(2s)^1(2p)^1$. To 0^{th} order, all of these configurations of the excited state would have the same energy. However, the spin-orbit interaction lifts part of that degeneracy.

Assume the simplest form of the spin-orbit interaction, $H_{S-O} = \gamma\, \mathbf{L} \cdot \mathbf{S}$, where $\mathbf{L}$ and $\mathbf{S}$ are the orbital and spin angular momentum operators respectively. Find the energy corrections to the $(1s)^2(2s)^1(2p)^1$ excited states of Be, and sketch an energy level diagram for these states. Be sure to indicate the degeneracy for each such level.

Hint: $\mathbf{L} \cdot \mathbf{S} = \frac{1}{2}(\mathbf{J}^2 - \mathbf{L}^2 - \mathbf{S}^2)$ where $\mathbf{J}$ is the total angular momentum operator.

Problem 2

A non-relativistic quantum particle of mass m is confined to a one-dimensional infinite square well potential with boundaries at coordinates $x = \pm a/2$.

a) Find the eigenfunctions ψ_n and eigenvalues E_n of the system's Hamiltonian.

b) The system is perturbed by an additional potential

$$V(x) = \epsilon \frac{2x}{a},$$

where ϵ is a small parameter. To first order in ϵ, determine the new ground state in the basis of the unperturbed eigenfunctions ψ_n.

c) Write the explicit form of the new perturbed ground state wave function that results if only the two largest-amplitude terms are kept in the result of **b**).

Problem 3

A mechanical one-dimensional harmonic oscillator of mass m and natural frequency ω is in an energy eigenstate $|n\!>$ (with $n = 0, 1, 2, ...$). Position and momentum are denoted by x and p respectively.

a) Calculate the quantum expectation values $<\!x\!>$, $<\!p\!>$, $<\!x^2\!>$, and $<\!p^2\!>$ for eigenstate $|n\!>$.

b) Check that the position-momentum uncertainty principle is satisfied for each state n.

c) Which energy eigenstate satisfies the *minimum*-uncertainty relation, i.e., minimizes the product $\Delta x \Delta p$?

Hint: The annihilation operator is

$$a \equiv \frac{1}{\sqrt{2\hbar m\omega}}(m\omega x + ip)$$

Problem 4

The entropy of a realistic gas with N molecules in a volume V at temperature T with total internal energy U is

$$S(U, V, N) = N k_B \ln\left(\frac{U}{N u_0} - a\frac{N}{V}\right)^{3/2} + N k_B \ln\left(\frac{V - bN}{V_0 - bN}\right) + \frac{3}{2} N k_B$$

where a, b, u_0, V_0 are positive constants. Here, b has the physical meaning of the volume occupied by an individual gas molecule, so that $V > bN$ is required. Also, assume that $V_0 > bN$.

a) Find the temperature as a function of V, N, and U.

b) Determine the pressure as a function of V, N, and T.

c) For a given N, V, and T, is the pressure in **b**) larger or smaller than that of an ideal gas? In particular, what happens to the pressure when $V \to bN$?

Problem 5

Assume that we have $N = 2$ non-interacting fermions in a system with the following single-particle energy level structure:

- Ground state: energy $E_1 = 0$, non-degenerate.
- Excited states: energies E_2, E_3 with $E_1 < E_2 < E_3$. Energy levels E_2 and E_3 are doubly degenerate.

Let N_i be the number of particles with energy E_i, so that $\sum_{i=1}^{3} N_i = N = 2$ for our two-particle system. This constraint allows 5 different combinations (N_1, N_2, N_3) when the degeneracies are taken into account. Assume the fermions are "spinless" such that there are no additional spin degrees of freedom.

a) In the table below, fill in these combinations and determine the resulting total energy $E_{\{N_i\}}$ of the 2-particle state. Note that the states of the system are characterized by five occupation numbers - one for each orbital. Fill in the number $G_{\{N_i\}}$ of *distinct states* giving rise to each combination (N_1, N_2, N_3).

N_1	N_2	N_3	$E_{\{N_i\}}$	$G_{\{N_i\}}$

b) Calculate the canonical partition function $Z(T)$ from the above table.

c) Assuming that $E_2 = \varepsilon$, $E_3 = 2\varepsilon$, calculate the average number of particles at energy E_2.

d) Assuming that $E_2 = \varepsilon$, $E_3 = 2\varepsilon$, calculate the average energy U in the canonical ensemble.

Problem 6

A simple model for the thermal disordering of alloys associates an energy with each atom not in its ideal position. Consider an alloy with two types of constituent atoms, called A and B. Assume there are N sites of type A and N sites of type B; the ideal configuration is that every A-site is occupied by an A atom, and every B-site is occupied by a B atom. In *all* allowed configurations, the numbers of A atoms and B atoms need not be equal but must add up to $2N$ so that all sites are filled.

An energy cost of ε is incurred if an A atom is on a B-site, and *vice versa*. Atoms of each type are indistinguishable.

a) Calculate the partition function for this model.

b) Calculate the Helmholtz free energy for this model and sketch it as a function of temperature, being sure to mark the appropriate scale on each axis.

c) Calculate the entropy S as a function of temperature and determine the limiting value of S as $T \to 0$. Is the result consistent with the third law of thermodynamics?