Exam #: ____________________

Printed Name: ____________________

Signature: ____________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Ph.D. Qualifying Examination
and
Master's Final Examination, PART I

Monday, March 31, 2008, 1:00 p.m. to 5:00 p.m.

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are eight equally weighted questions, each beginning on a new page. Read all eight questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic, and will be provided. **Personal calculators of any type are not allowed**. Paper dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries will not be allowed. No other papers or books may be used.**

When you have finished, come to the front of the room and hand your examination paper to the proctor; first put all problems in numerical order and staple them together.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand in your exam paper on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg ($0.511\,\mathrm{MeV/c^2}$)
Proton rest mass (m_p)	1.673×10^{-27} kg ($938\,\mathrm{MeV/c^2}$)
Neutron rest mass (m_n)	1.675×10^{-27} kg ($940\,\mathrm{MeV/c^2}$)
W^- rest mass (m_W)	$80.4\,\mathrm{GeV/c^2}$
Planck's constant (h)	$6.63 \times 10^{-34}\,\mathrm{J \cdot s}$
Speed of light in vacuum (c)	3.00×10^{8} m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Gravitational constant (G)	$6.67 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2}$
Permeability of free space (μ_0)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space (ϵ_0)	8.85×10^{-12} F/m
Mass of Earth (M_{Earth})	5.98×10^{24} kg
Mass of Moon (M_{Moon})	7.35×10^{22} kg
Mass of Sun (M_{Sun})	1.99×10^{30} kg
Radius of Earth (R_{Earth})	6.38×10^{6} m
Radius of Moon (M_{Moon})	1.74×10^{6} m
Radius of Sun (R_{Sun})	6.96×10^{8} m
Earth - Sun distance (R_{ES})	1.50×10^{11} m
Classical electron radius (r_0)	2.82×10^{-15} m
Gravitational acceleration on Earth (g)	$9.8\,\mathrm{m/s^2}$
Atomic mass unit	1.66×10^{-27} kg
One atmosphere (1 atm)	$1.01 \times 10^{5}\,\mathrm{N/m^2}$

Integrals

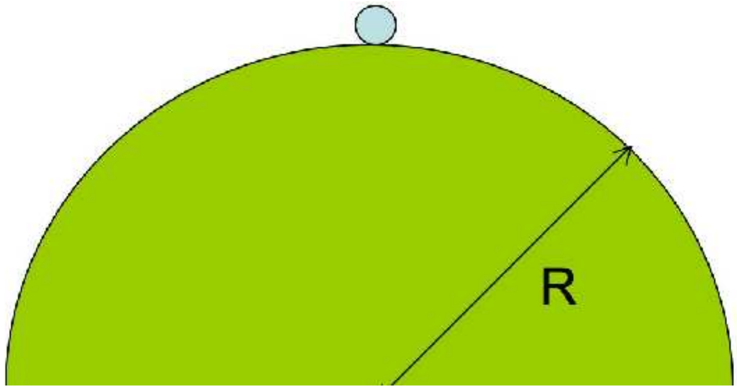
$$\int \frac{\mathrm{dx}}{\sqrt{\mathrm{x^2 \pm a^2}}} = \log\left(x + \sqrt{x^2 \pm a^2}\right) \qquad \text{(i)}$$

$$\int \frac{\mathrm{dx}}{(\mathrm{x^2 + a^2})} = \frac{1}{a}\tan^{-1}\frac{x}{a}, \text{ or } \frac{1}{a}\sin^{-1}\frac{x}{\sqrt{x^2 + a^2}} \qquad \text{(ii)}$$

$$\int \frac{\mathrm{dx}}{(\mathrm{x^2 \pm a^2})^{3/2}} = \frac{\pm x}{a^2\sqrt{x^2 \pm a^2}} \qquad \text{(iii)}$$

Problem 1

A solid ball of mass m and radius r lies on top of a hemisphere of radius R, which in turn is glued to a table. The ball starts from rest and rolls down without slipping until it leaves the surface. At what height (above the table surface) does the ball leave the surface?



Problem 2

In a collision of two particles of masses m_1 and m_2, the initial velocities are $\vec{u}_1 \neq 0$ and $\vec{u}_2 = \alpha \vec{u}_1$ with $\alpha \neq 0$. The initial kinetic energies of the two particles are equal.

(a) What value(s) can α have?

(b) Find the numerical value of m_1/m_2 such that m_1 will be at rest after the collision in case the collision is *totally inelastic*.

(c) Find the numerical value of m_1/m_2 such that m_1 will be at rest after the collision in case the collision is *elastic*.

Problem 3

A particle of mass m moves from (space, time) coordinate (0,0) to (X, T). For simplicity, assume both X and T are positive. Calculate the action of its path in the case of

(a) A free particle, for which $L = \frac{1}{2}m\dot{x}^2$.

(b) A particle in a potential $V(x) = m\epsilon x$ where ϵ is very small: give the result only to first order in ϵ.

Problem 4

A point particle of mass m in a uniform gravitational field is constrained to move on the surface of a sphere, centered at the origin. The radius $r(t)$ of this sphere is a given function of the time t. In the following, define the kinetic energy such that it accounts for all three Cartesian velocity components of the point particle in the lab frame.

(a) Obtain the Lagrangian for this constrained motion. For definiteness, start from spherical polar coordinates with the z-axis parallel to the gravitational field.

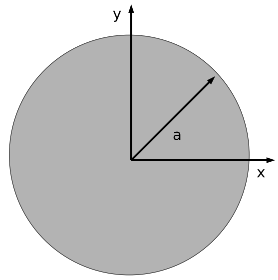
(b) Derive the Hamiltonian function H using the same coordinates as in (a).

(c) Calculate the total energy E of the particle. Is $E = H$?

(d) Use Hamilton's equations to identify at least one constant of motion other than the energy.

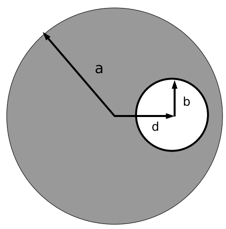
Problem 5

Consider an infinitely long, solid, cylindrical wire with radius a that has uniform current density $\vec{j}$, and carries a total current I. The wire is coaxial with the z-axis and the current flows in the +z direction.



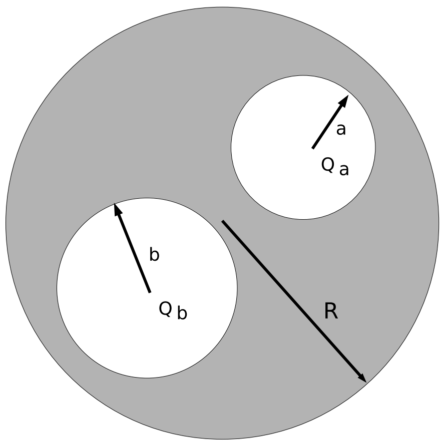
(a) Find the magnetic field $\vec{B}$ inside and outside the wire. Give your answer in cylindrical coordinates.

(b) Suppose the wire has a hole of radius b offset from the axis of the wire by distance d as shown in the figure ($b + d < a$). If the current density $\vec{j}$ is uniform outside the hole and has the same value as in part a, find the magnetic field $\vec{B}$ inside the hole. Give your result in Cartesian coordinates.



Problem 6

1. State Gauss's Law as an equation and concisely describe its concept in words.
2. Two spherical cavities of radii a and b are hollowed out from the interior of an initially neutral metal sphere of radius R. Point charges Q_a and Q_b are placed at the centers of the cavities as shown below.



   (a) Find the induced surface charge densities σ_a and σ_b on the surfaces of the two cavities. Briefly explain how you arrived at your solutions.

   (b) What is the electric field in each cavity as a function of position?

   (c) What are the net electric forces on Q_a and Q_b?

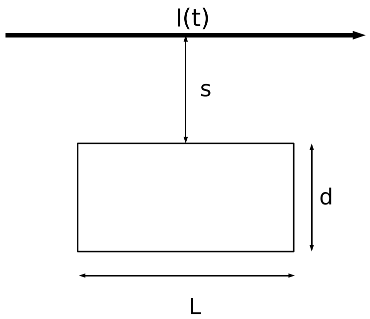
   (d) Find the induced surface charge density σ_R on the outer surface of the conductor. Briefly explain how you arrived at your solution.

Problem 7

A very long straight wire carries a current

$$I(t) = \alpha - \beta \, \mathrm{t} \tag{1}$$

during time interval $0 < \mathrm{t} < \tau$, where α and β are positive constants. A rectangular conducting loop is located near the wire, as shown below



(a) Find an expression for the induced current in the rectangular loop.

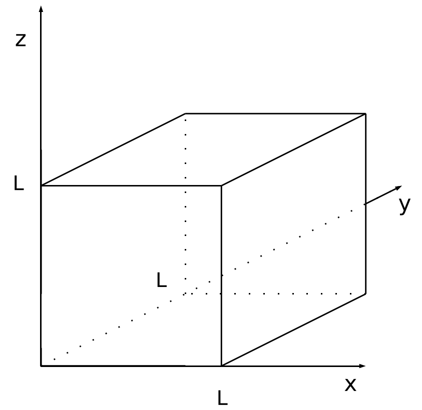
(b) What is the direction of the current in the loop?

(c) What is the force on the rectangular loop?

Note: Assume that the dimensions s, L, and d are small so that you may ignore time propagation effects in your calculations.

Problem 8

(a) Find the electric potential, Φ, inside a cube whose edges have length L, as shown in the figure.



The boundary conditions are:

(i) The potential vanishes on the four vertical surfaces and on the horizontal surface at $z = 0$.

(ii) On the horizontal surface at $z = L$ one has

$$\Phi = V_\circ \sin(\pi x/L)\, \sin(2\pi y/L) \tag{2}$$

where $V_\circ$ is some constant. (There are no charges inside the cube.)

(b) At the center of the cube calculate,

(i) the potential Φ

(ii) the electric field $\vec{E}$