The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are twelve equally weighted questions, each beginning on a new page. Read all twelve questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic, and will be provided. Personal calculators of any type are not allowed. Paper dictionaries may be used if they have been approved by the proctor before the examination begins. Electronic dictionaries will not be allowed. No other papers or books may be used.

When you have finished, come to the front of the room and hand your examination paper to the proctor; first put all problems in numerical order and staple them together.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand your exam paper in on time, an appropriate number of points may be subtracted from your final score.
### Constants

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron charge ($e$)</td>
<td>$1.60 \times 10^{-19}$ C</td>
</tr>
<tr>
<td>Electron rest mass ($m_e$)</td>
<td>$9.11 \times 10^{-31}$ kg (0.511 MeV/c²)</td>
</tr>
<tr>
<td>Proton rest mass ($m_p$)</td>
<td>$1.673 \times 10^{-27}$ kg (938 MeV/c²)</td>
</tr>
<tr>
<td>Neutron rest mass ($m_n$)</td>
<td>$1.675 \times 10^{-27}$ kg (940 MeV/c²)</td>
</tr>
<tr>
<td>$W^-$ rest mass ($m_{W}$)</td>
<td>80.4 GeV/c²</td>
</tr>
<tr>
<td>Planck's constant ($h$)</td>
<td>$6.63 \times 10^{-34}$ J · s</td>
</tr>
<tr>
<td>Speed of light in vacuum ($c$)</td>
<td>$3.00 \times 10^8$ m/s</td>
</tr>
<tr>
<td>Boltzmann’s constant ($k_B$)</td>
<td>$1.38 \times 10^{-23}$ J/K</td>
</tr>
<tr>
<td>Gravitational constant ($G$)</td>
<td>$6.67 \times 10^{-11}$ N · m²/kg²</td>
</tr>
<tr>
<td>Permeability of free space ($\mu_0$)</td>
<td>$4\pi \times 10^{-7}$ H/m</td>
</tr>
<tr>
<td>Permittivity of free space ($\varepsilon_0$)</td>
<td>$8.85 \times 10^{-12}$ F/m</td>
</tr>
<tr>
<td>Mass of Earth ($M_{Earth}$)</td>
<td>$5.98 \times 10^{24}$ kg</td>
</tr>
<tr>
<td>Mass of Moon ($M_{Moon}$)</td>
<td>$7.35 \times 10^{22}$ kg</td>
</tr>
<tr>
<td>Mass of Sun ($M_{Sun}$)</td>
<td>$1.99 \times 10^{30}$ kg</td>
</tr>
<tr>
<td>Radius of Earth ($R_{Earth}$)</td>
<td>$6.38 \times 10^6$ m</td>
</tr>
<tr>
<td>Radius of Moon ($R_{Moon}$)</td>
<td>$1.74 \times 10^6$ m</td>
</tr>
<tr>
<td>Radius of Sun ($R_{Sun}$)</td>
<td>$6.96 \times 10^8$ m</td>
</tr>
<tr>
<td>Earth - Sun distance ($R_{ES}$)</td>
<td>$1.50 \times 10^{11}$ m</td>
</tr>
<tr>
<td>Density of iron at low temperature ($\rho_{Fe}$)</td>
<td>$7.88 \times 10^3$ kg/m³</td>
</tr>
<tr>
<td>Classical electron radius ($r_0$)</td>
<td>$2.82 \times 10^{-15}$ m</td>
</tr>
<tr>
<td>Gravitational acceleration on Earth ($g$)</td>
<td>$9.8$ m/s²</td>
</tr>
<tr>
<td>Atomic mass unit</td>
<td>$1.66 \times 10^{-27}$ kg</td>
</tr>
<tr>
<td>One atmosphere (1 atm)</td>
<td>$1.01 \times 10^5$ N/m²</td>
</tr>
<tr>
<td>Specific heat of oxygen ($c_V$)</td>
<td>$21.1$ J/mole·K</td>
</tr>
<tr>
<td>Specific heat of oxygen ($c_p$)</td>
<td>$29.4$ J/mole·K</td>
</tr>
</tbody>
</table>

### Moments of Inertia


For a disk or cylinder of mass $M$ and radius $R$, about its symmetry axis: $(1/2)MR^2$.

For a solid sphere of mass $M$ and radius $R$, about any symmetry axis: $(2/5)MR^2$.

For a spherical shell of mass $M$ and radius $R$, about any symmetry axis: $(2/3)MR^2$. 
A solid sphere of uniform density and radius $r$ is rotating about a horizontal axis while sitting on a horizontal table. At time $t = 0$, the center of mass of the sphere is at rest with respect to the table, and the angular speed of rotation is $\omega_0$. If the coefficient of friction between the sphere and the table is $\mu$, and the acceleration of gravity is $g$, at what time does the sphere begin to roll on the table without slipping?
Problem 2

A bullet of mass $m$ is fired vertically into the air with an initial speed $v_0$ in a gravitational field of acceleration $g$. The bullet is subject to a quadratic drag force of magnitude $cv^2$.

a) Find an expression for the time required for the bullet to reach its maximum height.

b) Evaluate this expression for an initial velocity equal in magnitude to the terminal velocity of the bullet when dropped from a large height. You can assume $g$ is constant over this height.

c) Show that the time required for the particle to reach its maximum height approaches some finite limit as the initial velocity $v_0$ is increased without limit.
A mass $m$ attached to a spring of spring constant $k$ moves in one dimension on a table with a coefficient of friction $\mu$ in the presence of a vertical gravitational field of acceleration $g$.

a) Write down the equation(s) of motion for the mass.

b) If the mass starts at rest a distance $8\mu mg/k$ to the right of the equilibrium position of the spring, plot its subsequent displacement $x(t)$. 

\[\text{Diagram: Mass attached to a spring.} \]
An infinite horizontal slab with thickness $h$ has volume charge density

$$\rho(z) = \begin{cases} 
0 & z < 0 \\
2\rho_0 z/h & 0 < z < h \\
0 & h < z,
\end{cases}$$  \hspace{1cm} (1)

where $\rho_0$ is a normalization constant and $z$ is the vertical coordinate. There is no free charge outside the slab. A thin conducting sheet connected to ground lays on top of the slab. The permittivity of the material in the slab is the same as the vacuum ($\epsilon_0$).

a) What is the surface charge density on the grounded conductor?

b) Find the electric field everywhere.

c) Find the electric potential everywhere. Express your answer relative to ground.
Problem 5

A long coaxial cable carries current $I$. The current flows down the surface of the inner cylinder (radius $a$) and back along the outer cylinder (radius $b$). The region between the cylinders is filled by material with permeability $\mu_0$ and permittivity $\varepsilon_0$. Ignore effects of the ends of the cable.

a) Find the magnetic field $\vec{B}$ everywhere.

b) Find the energy stored per unit length of the cable.
Problem 6

A 6.0 μ-Henry inductor is placed in series with a 20 Ω resistor, and an emf, $\mathcal{E} = 3.0$ volts, is suddenly applied to the combination from a battery. The following questions refer to a time 0.3 μsec after the $\mathcal{E}$ is applied to the circuit.

a) What is the rate at which energy is delivered by the battery?

b) At what rate does energy appear as Joule heat in the resistor?

c) At what rate is energy stored in the magnetic field of the inductor?
Problem 7

We want to make an order of magnitude estimate of the mean free path of a molecule in a real gas.

a) Consider a gas consisting of hard spheres of radius $r$ and particle density $N/V$. Roughly how far can one sphere travel on average without bumping into another sphere?

b) Based on the considerations in a), obtain a rough numerical estimate of the mean free path $l_M$ of a gas (such as air) at standard temperature and pressure ($0 ^\circ C, 10^5$ Pa).
Problem 8

A gas is confined to a thermally insulated cylinder containing a partition. The volume on the left side of the partition is $V$ while the volume on the right side of the partition is $bV$. Each side contains $\nu$ moles of gas at temperature $T$. Recall that the expression for the entropy of a classical ideal gas is given by

$$S = N k_B \ln \left( \frac{V T^{3/2} \sigma_0}{N} \right)$$

where $\sigma_0$ is a universal constant.

a) Find the pressure in the two compartments.

The partition is now suddenly removed without doing work on the gas.

b) Find the total change in entropy if the gasses in the original volumes consist of the same type of atom.

c) Find the total change in entropy if the two gasses are different. Is the entropy greater or less than the result found for part b)? Explain the physical meaning of this result.
Problem 9

A one-dimensional harmonic oscillator has energy levels given by

\[ E_n = \hbar \omega (n + \frac{1}{2}) \]

with \( \omega \) being the oscillator frequency.

a) Calculate the canonical partition function \( Z \) for this oscillator.

b) From the partition function, calculate the internal energy \( U \) and the specific heat \( C_V \).
Problem 10

The electron in a hydrogen atom is initially in the $n = 2$ state. At some later time the electron undergoes a transition to the ground state by emitting a photon.

a) Assuming the atom is initially at rest and ignoring its recoil due to the emission, obtain the energy of the photon. Express your result in terms of the Rydberg energy.

b) To conserve energy and momentum, the atom must recoil. Assuming again the atom is initially at rest and using the photon energy you obtained in a), obtain the atom’s recoil velocity and its kinetic energy.

c) Now assume the atom is moving with initial velocity $\vec{v}$ along the direction of photon emission. Taking into consideration the atom’s recoil, obtain the energy of the emitted photon. Hint: Assume that the correction to the energy of the emitted photon is small.
Problem 11

A particle with mass $m$ is bound in a two-dimensional infinite square-well potential. The dimensions of the well along the $x$-direction and $y$-direction are $a$ and $b$, respectively. The potential energy inside the well equals zero, and is infinite everywhere outside the well.

a) Obtain the normalized energy eigenfunctions and the eigenvalues of this system.

b) Let the ratio of lengths be $a/b = 2$. Calculate the eigenenergies for the six lowest-energy eigenfunctions, pointing out any degeneracies that occur.
Problem 12

In a reference frame \( O \), two spacetime events have coordinates \((x_1, t_1)\) and \((x_2, t_2)\). In a second reference frame \( O' \), moving at constant velocity \( v \) in the \( x \)-direction with respect to \( O \), the events have coordinates \((x'_1, t'_1)\) and \((x'_2, t'_2)\), respectively.

a) What condition must the coordinates satisfy so that a frame \( O' \) exists in which the two events occur at the same spatial point? Obtain the velocity \( v \) of \( O' \) with respect to \( O \) in this case.

b) Does there exist a value for \( v \) such that the two events occur \textit{simultaneously} in the \( O' \) frame? Under what condition?