Exam #: ________________________

Printed Name: ________________________

Signature: ________________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Master's Final Examination

and

Ph.D. Qualifying Examination, PART I

Monday, April 3, 2006, 1:00 p.m. to 5:00 p.m.

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are twelve equally weighted questions, each beginning on a new page. Read all twelve questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic. **Calculators with stored equations or text are not allowed**. Paper dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries will not be allowed. No other papers or books may be used.**

When you have finished, come to the front of the room and hand your examination paper to the proctor; first put all problems in numerical order and staple them together.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand your exam paper in on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg (0.511 MeV/c^2)
Proton rest mass (m_p)	1.673×10^{-27} kg (938 MeV/c^2)
Neutron rest mass (m_n)	1.675×10^{-27} kg (940 MeV/c^2)
W^- rest mass (m_W)	80.4 GeV/c^2
Planck's constant (h)	6.63×10^{-34} J$\cdot$ s
Speed of light in vacuum (c)	3.00×10^{8} m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Gravitational constant (G)	6.67×10^{-11}N$\cdot$ m^2/kg^2
Permeability of free space (μ_0)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space (ϵ_0)	8.85×10^{-12} F/m
Mass of Earth ($M_{\rm Earth}$)	5.98×10^{24} kg
Mass of Moon ($M_{\rm Moon}$)	7.35×10^{22} kg
Radius of Earth ($R_{\rm Earth}$)	6.38×10^{6} m
Radius of Moon ($M_{\rm Moon}$)	1.74×10^{6} m
Radius of Sun ($R_{\rm Sun}$)	6.96×10^{8} m
Earth - Sun distance ($R_{\rm ES}$)	1.50×10^{11} m
Density of iron at low temperature ($\rho_{\rm Fe}$)	7.88×10^{3} kg/m^3
Classical electron radius (r_0)	2.82×10^{-15} m
Gravitational acceleration on Earth (g)	9.8 m/s^2
Atomic mass unit	1.66×10^{-27} kg
Specific heat of oxygen (c_V)	21.1 J/mole$\cdot$ K
Specific heat of oxygen (c_P)	29.4 J/mole$\cdot$ K

Moments of Inertia

For a hoop of mass M and radius R, about its symmetry axis: MR^2.

For a disk or cylinder of mass M and radius R, about its symmetry axis: $(1/2)\,MR^2$.

For a solid sphere of mass M and radius R, about any symmetry axis: $(2/5)\,MR^2$.

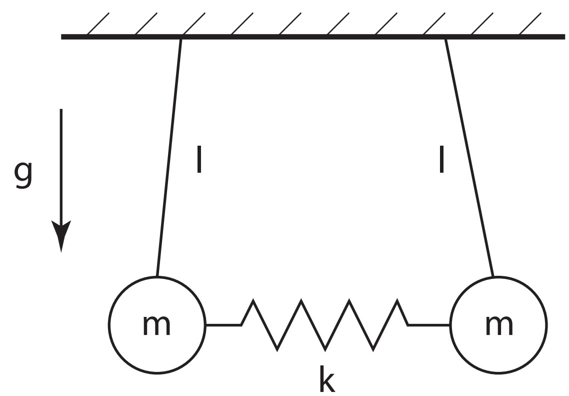
For a spherical shell of mass M and radius R, about any symmetry axis: $(2/3)\,MR^2$.

Problem 1

A uniform sphere of mass M and radius R is given an initial velocity v_0 along a smooth, horizontal surface. The sphere, which is not rotating initially, slides along the surface for a distance s before it begins to roll without slipping. Assuming a vertical gravitational acceleration g, find an expression for the coefficient of kinetic friction μ_k

Problem 2

Two identical pendulums of length l and mass m hanging in a vertical gravitational field g are connected by a horizontal spring with spring constant k as shown below. Assuming small amplitude oscillations, the motion of the pendulums is constrained to be horizontal along the line of the spring (i.e. longitudinal oscillations).



a) Briefly explain why this system has two normal modes of oscillation.

b) Briefly describe the motion of each normal mode.

c) Find the oscillation frequencies of the normal modes.

Problem 3

An ideal, uniform spring of mass M and spring constant k has a length L_0 when no external forces are applied. A mass m is attached to one end, and the spring is suspended from the other end in a uniform gravitational field with acceleration g. Find the total equilibrium length of the hanging spring.

Problem 4

The current density in a long straight cylindrical wire of radius R oriented in the $\hat{z}$ direction is proportional to the distance, r, from the axis: $\vec{j} = kr\hat{z}$, where k is a constant.

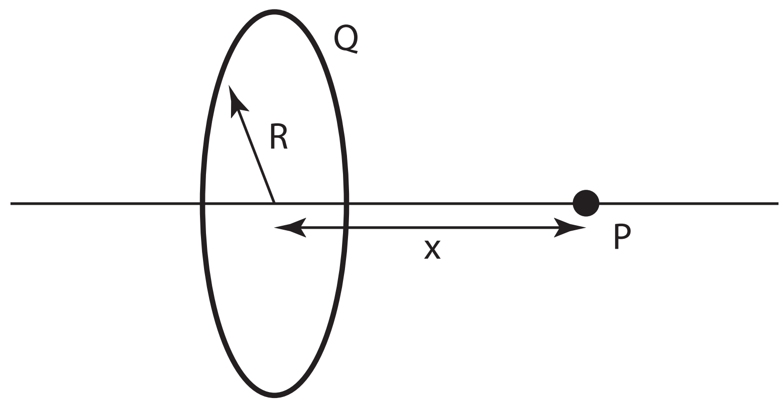
a) Find the total current, $\vec{J}$, in the wire.

b) Find the direction and magnitude of the magnetic field, $\vec{B}$, inside and outside the wire as a function of r. (Assume the magnetic permeability inside the wire has the value $\mu = \mu_0$.)

c) Sketch the magnitude of the magnetic field as a function of r.

Problem 5

Consider a thin, non-conducting circular loop of radius R that has a total charge $Q > 0$ uniformly distributed over it as shown below.



a) Find the electric potential at point P on the symmetry axis a distance x away from the center of the ring.

b) A small particle of mass m and charge $q < 0$ is placed at point P and released. If $R \gg x$, the particle will undergo oscillations along the axis of symmetry. Find the angular frequency of the oscillations.

Problem 6

A long, thin solenoid is formed by winding 100 turns of thin wire around a solid cylinder of radius 1 mm and length 10 mm. Assume the cylinder is an insulator and has $\mu = \mu_0$.

a) Ignoring edge effects near the two ends of the solenoid, find the magnitude of the magnetic field inside the cylinder when a 10 mA current flows through the wire.

b) Approximately what is the magnitude of the induced voltage that will be measured across the solenoid if it is immersed in a changing magnetic field with $dB/dt = 1$ T/s? Assume the direction of B is along the axis of the solenoid and that a high impedance voltmeter is used.

c) What is the self-inductance of the coil?

Problem 7

An air conditioning unit cools the interior of a building by extracting heat from the air inside at 20°C and exhausting it to the air outside at 40°C. (The air inside the building and the air outside the building can both be treated as heat reservoirs.)

Calculate the maximum rate at which heat can be extracted from the interior if the heat pump is driven by a 1 kW motor.

Problem 8

Consider a system of N non-interacting magnetic dipoles with dipole moment μ in a magnetic field B at thermal equilibrium with a heat bath of temperature T.

a) Sketch a rough graph of how you would conceptually expect the heat capacity at constant magnetic field, C_B, to behave as a function of temperature. Briefly explain the physics controlling the limiting behavior of C_B at very low temperature ($k_B T \ll \mu B$) and at very high temperature ($k_B T \gg \mu B$).

b) Write the partition function and use it to calculate $\langle E \rangle$, the mean energy of the spin system, as a function of μ, B, and T.

c) Find an explicit expression for C_B and verify that it agrees with your reasoning in part a).

Problem 9

The plutonium aboard NASA's *New Horizons* Pluto mission does **not** serve any propulsion-related purposes, but nuclear-powered spacecraft have been discussed since the earliest days of nuclear technology. Let's find out why. Consider a rocket motor whose exhaust consists of molecules of mass m at a temperature T.

a) What is the root-mean-square velocity of these molecules?

b) The rocket mass is M and the starting fuel has mass $M_0 - M$, such that the total liftoff mass is M_0. Assume the average velocity of the exhaust molecules is w. Show that, in the absence of external forces, the terminal velocity of a single-stage rocket will be

$$v = w \ln(M_0/M).$$

c) Combining these results, what type of molecule would give the highest terminal velocity for fixed mass and temperature?

d) What is the advantage, then, of a nuclear reactor over a more traditional chemical reaction?

Problem 10

Consider the collision shown above. A particle is at rest with energy E_0. A photon of energy $h\nu$ is incident upon the particle. After the collision the photon is scattered by some angle θ and the particle is given some momentum.

a) Give an argument why the energy of the photon after the collision must be less than the photon energy prior to the collision.

b) Derive an expression for the change in wavelength of the photon that shows that this change in wavelength is independent of the original photon energy.

Problem 11

A common treatment for brain tumors involves irradiating the tumor with gamma rays. Scattering of the gamma rays within the brain must be sufficiently low for them to reach the site of the tumor. In a simple model, the brain can be approximated as a volume of water, and the gamma rays scatter primarily off the electrons in the water molecules.

a) Picture the gamma ray photons as point particles and the electrons as hard spheres. Calculate the geometric cross-section of the electron spheres as a function of the sphere radius.

b) Find the number of electrons per unit volume.

c) The radius of the electron sphere can be estimated by assuming that each electron is a uniformly-charged spherical shell, and the energy stored in the associated electric field equals the electron rest energy $E = m_e c^2$. Find this radius.

d) Use your previous answers to calculate the mean free path of a photon traveling through the brain in this simple model.

e) What fraction of incident photons will not have been scattered after traversing 8 cm of brain?

Problem 12

In 1987, a supernova in the Large Magellanic Cloud produced a burst of neutrinos and anti-neutrinos which were detected in large underground detectors filled with water. In these detectors, one of the observable reactions which will produce a signal is

$$\overline{\nu}_e + p \rightarrow n + e^+$$

where p is a proton in the water. The resulting positron is detected by its emission of Cherenkov light, which occurs when a charged particle travels faster than the speed of light in the medium.

a) Using 1.33 for the index of refraction of water, determine the minimum kinetic energy of the positron which could produce Cherenkov light.

b) Determine the minimum neutrino energy which could give rise to a Cherenkov signal. (Assume that both the neutrino rest mass and the neutron recoil kinetic energy are zero.)