Exam #: ______________________

Printed Name: ______________________

Signature: ______________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Ph.D. Qualifying Examination
and
Master's Final Examination, PART I

Monday, 29 September 2008, 9:00 a.m. to 1:00 p.m.

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are eight equally weighted questions, each beginning on a new page. Read all eight questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic, and will be provided. **Personal calculators of any type are not allowed**. Paper dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries will not be allowed. No other papers or books may be used.**

When you have finished, come to the front of the room and hand your examination paper to the proctor; first put all problems in numerical order and staple them together.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand in your exam paper on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg (0.511 MeV/c^2)
Proton rest mass (m_p)	1.673×10^{-27} kg (938 MeV/c^2)
Neutron rest mass (m_n)	1.675×10^{-27} kg (940 MeV/c^2)
W^- rest mass (m_W)	80.4 GeV/c^2
Planck's constant (h)	6.63×10^{-34} J$\cdot$ s
Speed of light in vacuum (c)	3.00×10^{8} m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Gravitational constant (G)	6.67×10^{-11}N$\cdot$ m^2/kg^2
Permeability of free space (μ_0)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space (ϵ_0)	8.85×10^{-12} F/m
Mass of Earth ($M_\oplus$)	5.98×10^{24} kg
Mass of Moon ($M_{\rm Moon}$)	7.35×10^{22} kg
Mass of Sun ($M_\odot$)	1.99×10^{30} kg
Radius of Earth ($R_\oplus$)	6.38×10^{6} m
Radius of Moon ($M_{\rm Moon}$)	1.74×10^{6} m
Radius of Sun ($R_\odot$)	6.96×10^{8} m
Earth - Sun distance ($R_{\oplus,\odot}$)	1.50×10^{11} m
Classical electron radius ($r_\circ$)	2.82×10^{-15} m
Gravitational acceleration on Earth (g)	9.8 m/s^2
Atomic mass unit	1.66×10^{-27} kg
One atmosphere (1 atm)	1.01×10^{5} N/m^2

Laplacian

$$\nabla^2\psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}$$

Problem 1

A uniform thin rod of mass M and length L lies at rest on a frictionless surface. A ball of putty of mass m moves along the surface with velocity v perpendicular to the rod. The putty strikes the rod at a point $L/4$ below the upper end and sticks to the rod.

a) Determine the center of mass (CM) position of the rod-putty system right after the collision.

b) What is the velocity of the CM after the collision?

c) Calculate what fraction of translational kinetic energy of the rod-putty system is lost due to the collision. Where did this energy go?

d) In the special case that $m = M$, calculate the angular velocity around the center of mass after the collision.

Problem 2

A rock of mass $m = 70.0$ kg is dropped straight down from a height $h = 70.0$ m.

a) Calculate the speed with which the rock hits the ground (ignoring friction) in three different ways:

   I. Within Newtonian physics, approximating the initial potential gravitational energy of the rock by mgh, where g is the gravitational acceleration on the surface of Earth;

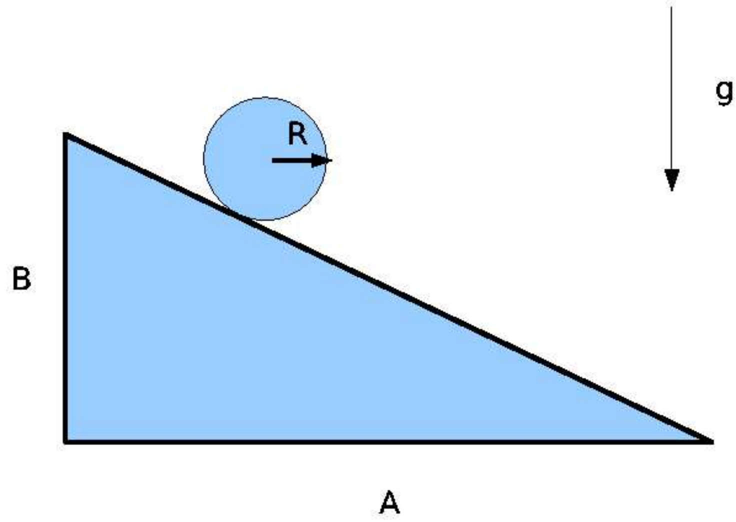
   II. Within Newtonian physics, but modeling the Earth as a spherically symmetric non-rotating mass distribution; and

   III. Using Special Relativity (so that the kinetic energy of mass m is $K = [\gamma - 1]mc^2$) and assuming mgh for the initial potential energy of the rock.

b) Calculate the *relative difference* between answers (II) and (I), and that between answers (III) and (I).

Problem 3

A solid ball with mass M and radius R rolls (without slipping) down an inclined plane with angle $\theta = \tan^{-1}$ (B/A). The mass M starts from rest at height h. Gravity, $\vec{g}$, points downward as indicated on the figure.



a) Calculate the magnitude of the friction force that acts on the ball.

b) Calculate the angular momentum L (around its center of mass) with which the ball arrives at the bottom of the incline.

Problem 4

Consider the Lagrangian

$$L = \frac{1}{2}m\dot{\vec{r}}^2 + q\dot{\vec{r}} \cdot \vec{A}(\vec{r}, t) - q\phi(\vec{r}, t),$$

for a particle with charge q and mass m. $\vec{A}$ and ϕ are the vector potential and the scalar potential, respectively, and they are assumed to be given functions of $\vec{r}$ and t

a) Write down the Euler-Lagrange equation for the dynamical variable $\vec{r}$, in terms of $\vec{A}$ and ϕ.

b) Rewrite the equation from part (a) in terms of $\vec{E}$ and $\vec{B}$ defined by

$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t},$$

and

$$\vec{B} = \vec{\nabla} \times \vec{A}.$$

You may want to use the vector identity:

$$\vec{\nabla}(\vec{a} \cdot \vec{b}) = (\vec{a} \cdot \vec{\nabla})\vec{b} + (\vec{b} \cdot \vec{\nabla})\vec{a} + \vec{a} \times (\vec{\nabla} \times \vec{b}) + \vec{b} \times (\vec{\nabla} \times \vec{a}).$$

Comment on the final form of the equation of motion.

Problem 5

Two infinitely thin concentric conducting spherical shells have radii R_1 and R_2, where $R_1 < R_2$. The total charge on the inner shell is $3Q$, and the total charge on the outer shell is $-5Q$. Calculate:

a) the magnitude and direction of the electric field, $\vec{E}$, and

b) the total energy of the electric field

inside each of the following regions:

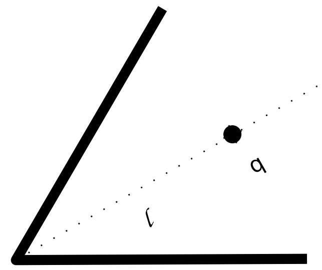
i) $r < R_1$,

ii) $R_1 < r < R_2$,

ii) $R_2 < r$.

Problem 6

Two perfectly conducting, infinite half-planes intersect at angle $\pi/3$ (see figure.) A charge q is located half-way between the planes at distance ℓ from the intersection. Using the method of image charges, calculate how much work was required to bring the charge q from infinity to its present position following the dotted line.



Problem 7

A small antenna with dipole moment $\vec{p}$ is located in vacuum at the origin of the coordinate frame, and emits electric dipole radiation of frequency ω. The electric field observed at position $\vec{r}$, at time t is:

$$\vec{E}(\vec{r},t) = \left\{ k^2 \left[(\hat{r} \times \vec{p}) \times \hat{r}\right] \frac{e^{ikr}}{r} + e^{ikr} \left(\frac{1}{r^3} - i\frac{k}{r^2} \right) \left[3\hat{r}\,(\hat{r}\cdot\vec{p}) - \vec{p}\right] \right\} e^{-i\omega t}.$$

Here, $k = \omega/c$, $\hat{r}$ is the unit vector parallel to $\vec{r}$, and $\vec{p}$ is the dipole moment of the antenna.

a) Find the expression for the $\vec{E}$ field emitted parallel to $\vec{p}$ (*i.e.*, at points where $\vec{r}$ is parallel to $\vec{p}$).

b) Rewrite the $\vec{E}$ field expression found in part (a) in the form

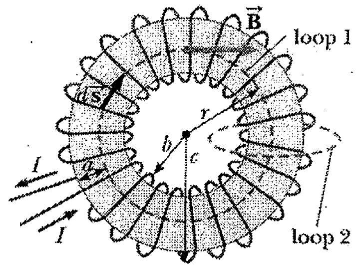
$$\vec{E}(\vec{r},t) = \hat{r}\; f(k,\vec{r})\; e^{i(\varphi(k,\vec{r})-\omega t)},$$

where $\hat{r}$ is the radial unit vector, $f(k,\vec{r})$ is a real, scalar function and $\varphi(k,\vec{r})$ is a phase factor.

c) The phase velocity of the $\vec{E}$ field is $\vec{v}_p$. Calculate $\vec{v}_p$ in the direction parallel to $\vec{p}$. What is $\vec{v}_p$ in the limit $r \to \infty$?

Problem 8

A toroid consists of a conducting wire wrapped around a ring made of a nonconducting material as shown in the Figure. Loop 1 is in the plane of the Figure and Loop 2 is perpendicular to the plane of the Figure.



a) For a toroid having N closely spaced turns of wire, calculate the magnetic field in the region occupied by the torus a distance r from the center.

b) Under what conditions can the magnetic field inside the torus be considered to have approximately uniform magnitude?

c) Argue that the magnetic field outside the torus is not zero.

d) What is the direction of the magnetic field at the center of the torus ($i.e.$, $r = 0$)?