Exam #: ________________________

Printed Name: ________________________

Signature: ________________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Ph.D. Qualifying Examination
and
Master's Final Examination, PART I

Tuesday, September 11, 2007, 1:00 p.m. to 5:00 p.m.

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are eight equally weighted questions, each beginning on a new page. Read all eight questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic, and will be provided. **Personal calculators of any type are not allowed**. Paper dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries will not be allowed. No other papers or books may be used.**

When you have finished, come to the front of the room and hand your examination paper to the proctor; first put all problems in numerical order and staple them together.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand your exam paper in on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg (0.511 MeV/c^2)
Proton rest mass (m_p)	1.673×10^{-27} kg (938 MeV/c^2)
Neutron rest mass (m_n)	1.675×10^{-27} kg (940 MeV/c^2)
W^- rest mass (m_W)	80.4 GeV/c^2
Planck's constant (h)	6.63×10^{-34} J$\cdot$ s
Speed of light in vacuum (c)	3.00×10^{8} m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Gravitational constant (G)	6.67×10^{-11}N$\cdot$ m^2/kg^2
Permeability of free space (μ_0)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space (ϵ_0)	8.85×10^{-12} F/m
Mass of Earth (M_{Earth})	5.98×10^{24} kg
Mass of Moon (M_{Moon})	7.35×10^{22} kg
Mass of Sun (M_{Sun})	1.99×10^{30} kg
Radius of Earth (R_{Earth})	6.38×10^{6} m
Radius of Moon (M_{Moon})	1.74×10^{6} m
Radius of Sun (R_{Sun})	6.96×10^{8} m
Earth - Sun distance (R_{ES})	1.50×10^{11} m
Classical electron radius (r_0)	2.82×10^{-15} m
Gravitational acceleration on Earth (g)	9.8 m/s^2
Atomic mass unit	1.66×10^{-27} kg
One atmosphere (1 atm)	1.01×10^{5} N/m^2

Laplacian operator

Cartesian coordinates:

$$\nabla^2\psi = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}$$

Cylindrical coordinates:

$$\nabla^2\psi = \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial\psi}{\partial r}) + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\phi^2} + \frac{\partial^2\psi}{\partial z^2}$$

Spherical coordinates:

$$\nabla^2\psi = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial\psi}{\partial r}) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial\psi}{\partial\theta}) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}$$

Spherical harmonics

The spherical harmonics Y_{lm} have the normalization property

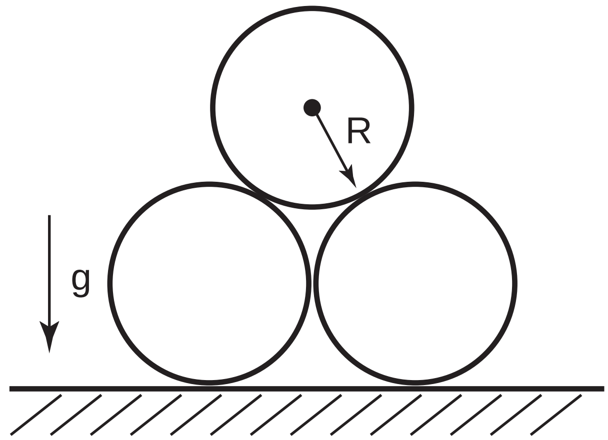
$$\int_{-1}^{1} d\cos\theta \int_{0}^{2\pi} d\phi \; Y_{lm}^{\star}(\theta,\phi)\, Y_{l'm'}(\theta,\phi) = \delta_{ll'}\,\delta_{mm'}.$$

The first few are

$$\begin{aligned}
Y_{00} &= \frac{1}{\sqrt{4\pi}} \\
Y_{11} &= -\sqrt{\frac{3}{8\pi}}\,\sin\theta\, e^{i\phi} \\
Y_{10} &= \sqrt{\frac{3}{4\pi}}\,\cos\theta \\
Y_{22} &= \sqrt{\frac{15}{32\pi}}\,\sin^2\theta\, e^{2i\phi} \\
Y_{21} &= -\sqrt{\frac{15}{8\pi}}\,\sin\theta\,\cos\theta\, e^{i\phi} \\
Y_{20} &= \sqrt{\frac{5}{16\pi}}\,[3\cos^2\theta - 1]
\end{aligned}$$

with $Y_{l,-m}(\theta,\phi) = -Y_{lm}^{*}(\theta,\phi)$.

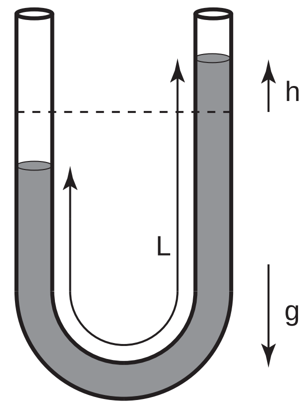
Problem 1



Three identical wooden cylinders of mass m and radius R are placed on a horizontal cement floor in a uniform gravitational field of acceleration g as shown. Find the minimum value of the coefficient of static friction μ between the cylinders and the floor such that there is neither translational nor rotational movement.

Problem 2

A non-viscous, incompressible fluid of density ρ is in a U-tube of constant cross-sectional area A. The variable h is the vertical displacement of the fluid on the right side of the U-tube from the equilibrium fluid level. The total length of fluid in the tube is L, and the fluid moves under a constant gravitational field of acceleration g.



a) Write down the total potential energy of the fluid $U(h)$.

b) Write down the kinetic energy of the fluid $T(h, \dot{h})$.

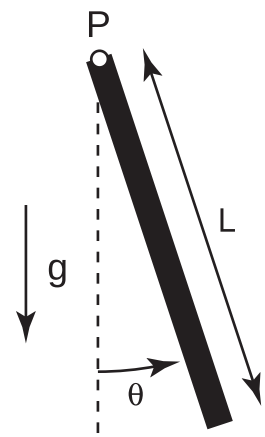
c) Write down the Lagrangian of the fluid $\mathcal{L}(h, \dot{h})$.

d) Derive the equation of motion for h from this Lagrangian.

e) If the fluid is initially at rest with $h(t = 0) = h_0$, find $h(t)$ for all later times t.

Problem 3

A uniform rod of length L and mass m is suspended from one end at point P in a uniform gravitational field with acceleration g. Assume frictionless motion constrained to a vertical plane.



a) When the rod makes an angle θ from the vertical, as shown, calculate the torque τ about the axis normal to the rotation plane through the support point P.

b) Show that for $\theta \ll 1$, the equation of motion for θ is that of a harmonic oscillator.

c) Determine an expression for the period of oscillations.

Problem 4

Consider a classical particle of mass m subject to a potential energy

$$V(x) = -\alpha x^2 e^{-\beta x},$$

where α and β are positive real constants and x is the position of the particle.

a) Sketch $V(x)$. You needn't label any of the points on the axes, just get the shape right. You may find it helpful to first find the maxima and minima, and to think about the behavior of V for $x \to \pm\infty$.

b) Find the position(s) of stable equilibrium.

c) Find the angular frequency(ies) of small oscillations about the equilibrium position(s).

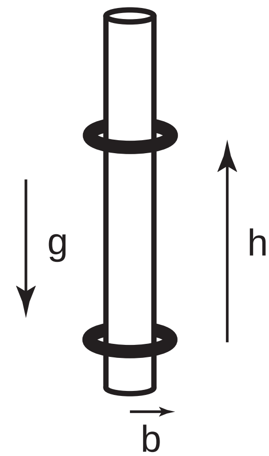
Problem 5

Linearly polarized light of wavelength λ is incident normally on a crystalline, plane parallel plate of thickness d. Waves polarized along the two orthogonal optical axes lying in the plane of the plate travel at velocities c/n_1 and c/n_2.

a) For what values of d can the transmitted light be circularly polarized?

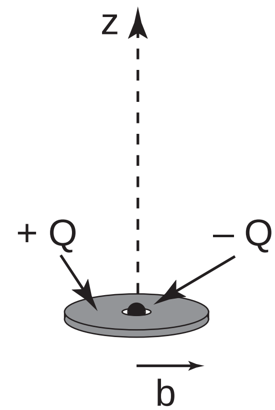
b) For what incident polarization angles with respect to the optical axes will the transmitted light be circularly polarized?

Problem 6



Two rings each of mass m and radius b have magnetic dipole moments of magnitude μ. The rings are placed on a nonconducting, frictionless pole aligned with the $\hat{z}$ direction. Gravity is given by $\vec{g} = -g\hat{z}$. The dipole moment for the bottom ring points in the $+\hat{z}$ direction and the dipole moment for the top ring points in the $-\hat{z}$ direction. The bottom ring is fixed to the pole while the top ring is free to slide up and down the pole. Find the equilibrium height h at which the top ring *floats* above the bottom ring. Assume that $b \ll h$.

Problem 7



A uniformly charged thin disk of total charge $+Q$ and radius b has a central point charge of -Q fixed at its center. The disk and the point charge lie in the x-y plane.

a) Find the electric potential for a field point located on the z-axis.

b) Find the electric potential everywhere to leading order in b/r, where the distance from the center of the disk to the field point is $r \gg b$.

Problem 8

Consider a long, cylindrical, non-magnetic wire of radius a carrying a steady current I. The conductivity of the wire is σ.

a) Find the Poynting vector inside and just outside the wire.

b) Calculate the flux of energy through the surface of a cylinder of radius r inside the wire.

c) Compare the flux of energy in part b) with the Joule heat produced by the current inside the cylinder and comment on your result.