Exam #: ______________________

Printed Name: ______________________

Signature: ______________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Master's Final Examination
and
Ph.D. Qualifying Examination, PART I

Wednesday, September 13, 2006, 1:00 p.m. to 5:00 p.m.

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are twelve equally weighted questions, each beginning on a new page. Read all twelve questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic, and will be provided. **Personal calculators of any type are not allowed**. Paper dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries will not be allowed. No other papers or books may be used.**

When you have finished, come to the front of the room and hand your examination paper to the proctor; first put all problems in numerical order and staple them together.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand your exam paper in on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg (0.511 MeV/c^2)
Proton rest mass (m_p)	1.673×10^{-27} kg (938 MeV/c^2)
Neutron rest mass (m_n)	1.675×10^{-27} kg (940 MeV/c^2)
W^- rest mass (m_W)	80.4 GeV/c^2
Planck's constant (h)	6.63×10^{-34} J$\cdot$ s
Speed of light in vacuum (c)	3.00×10^8 m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Gravitational constant (G)	6.67×10^{-11}N$\cdot$ m^2/kg^2
Permeability of free space (μ_0)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space (ϵ_0)	8.85×10^{-12} F/m
Mass of Earth (M_{Earth})	5.98×10^{24} kg
Mass of Moon (M_{Moon})	7.35×10^{22} kg
Mass of Sun (M_{Sun})	2.0×10^{30} kg
Radius of Earth (R_{Earth})	6.38×10^6 m
Radius of Moon (M_{Moon})	1.74×10^6 m
Radius of Sun (R_{Sun})	7.0×10^8 m
Earth - Sun distance (R_{ES})	1.50×10^{11} m
Density of iron at low temperature (ρ_{Fe})	7.88×10^3 kg/m^3
Classical electron radius (r_0)	2.82×10^{-15} m
Gravitational acceleration on Earth (g)	9.8 m/s^2
Atomic mass unit	1.66×10^{-27} kg
One atmosphere (1 atm)	1.01×10^5 N/m^2
Specific heat of oxygen (c_V)	21.1 J/mole$\cdot$ K
Specific heat of oxygen (c_P)	29.4 J/mole$\cdot$ K

Moments of Inertia

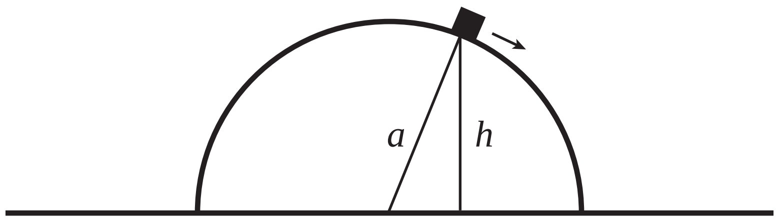
For a hoop of mass M and radius R, about its symmetry axis: MR^2.

For a disk or cylinder of mass M and radius R, about its symmetry axis: $(1/2)\,MR^2$.

For a solid sphere of mass M and radius R, about any symmetry axis: $(2/5)\,MR^2$.

For a spherical shell of mass M and radius R, about any symmetry axis: $(2/3)\,MR^2$.

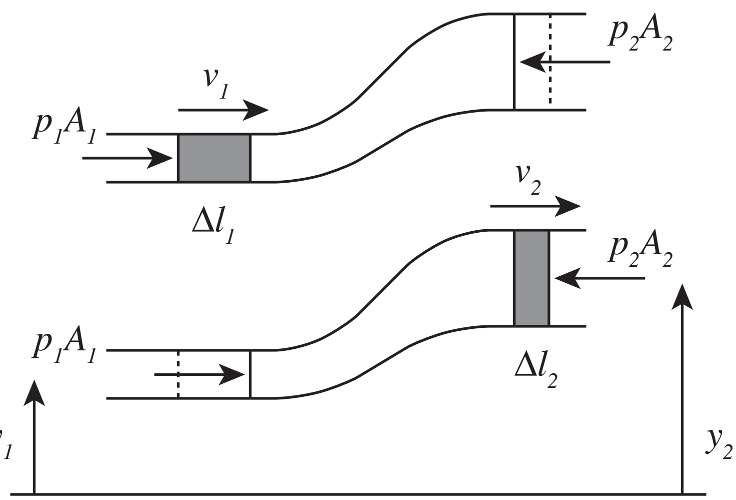
Problem 1



A particle is placed at the top of a smooth, frictionless, hemispherical surface of radius a. If the particle is disturbed slightly it will slide off the surface. At what height h relative to the radius of the sphere will the particle leave the surface?

Problem 2

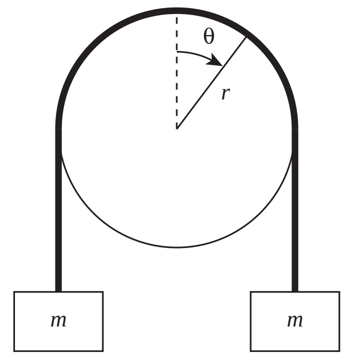
Consider the work done on a portion of moving fluid (dark-shaded region below) from one end of a variable cross-section pipe to the other against pressures p_1 and p_2 as shown. Note that the fluid is lifted a distance $y_2 - y_1$ in a gravitational field of acceleration g. Assume that the fluid is incompressible.



a) Write down an expression for the work done (W) in moving the portion up the pipe.

b) Use the work-energy theorem to derive Bernoulli's equation from part a). In other words, find a function of pressure, density, velocity, and height which is invariant along the pipe.

Problem 3



Two stationary masses, each of mass m, are connected by a massless belt of width w. The masses are suspended over a cylinder of radius r in a gravitational field with acceleration g. Find the magnitude and direction of the pressure exerted by the belt on the cylinder as a function of the polar angle θ.

Problem 4

Consider a pair of very long, very thin, parallel wires that are suspended a distance d apart. To identify the wires we denote them as A and B.

a) Wire A carries a DC current of magnitude I_A. Derive an expression for the magnetic field outside the wire as a function of the radial distance r from the wire center.

b) Wire B carries a DC current of magnitude I_B. Find an expression for the force per unit length on wire B due to the magnetic field created by wire A.

c) Sketch the wires, choose specific directions for currents I_A and I_B, and indicate the directions and relative magnitude of the forces felt by wires A and B.

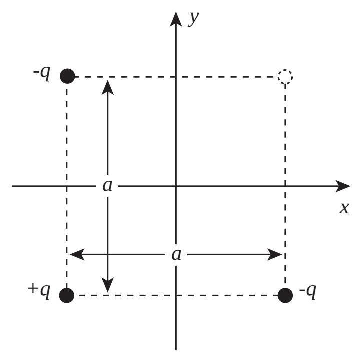
Problem 5

Consider a free electron traversing a region of space filled with a cold gas where the electric field is given by $\vec{E} = -E_0\hat{x}$ and the magnetic field is given by $\vec{B} = B_0\hat{z}$. E_0 and B_0 are positive constants. Due to collisions of the electron with the gas atoms, after some time the mean drift velocity of the electron is given by $\vec{v} = \frac{\mu}{e}\vec{F}$ where μ is a constant called the mobility, $\vec{F}$ is the Lorentz force on the electron, and e is the magnitude of the electron charge. Find the components of the electron's mean drift velocity in the gas-filled region (ignoring any small thermal fluctuations). Find $\tan\theta = v_y/v_x$. Does it depend on E_0?

Problem 6

Three charges are situated at the corners of a square of side a in the x-y plane as shown.

a) How much work does it take to bring in another charge, $+q$, from far away and place it in the fourth corner.



b) How much work does it take to assemble the entire configuration of four charges?

c) Suppose infinite conducting planes are placed along the axes in the figure above (that is, in the x-z and y-z planes, intersecting through the origin). Using your answer for part b), and without further calculation, deduce the amount of work required to bring a single point charge, $+q$, from far away to the point $(x, y, z) = (a/2, a/2, 0)$ with no other charges present. Briefly explain your reasoning.

Problem 7

A piezoelectric substance is one where a mechanical deformation produces an electric polarization, and vice versa. Consider a 1D deformation of such a material. The stress σ (force per unit area) in the x-direction is found from experiment to produce a polarization P proportional to σ:

$$\alpha \equiv \left.\frac{\partial P}{\partial \sigma}\right|_{E,T}$$

where α is a constant, T is the temperature, and E is the electric field. For this system, the differential energy change is

$$dU = TdS - \sigma A dL + EdP$$

where S is the entropy, A is the cross-sectional area, and L is the length of the sample.

a) Derive an expression for $\beta \equiv \left.\frac{\partial L}{\partial E}\right|_{\sigma,T}$.

b) Suppose a substance has $\alpha > 0$. Will the substance expand or contract upon application of an electric field? Explain your reasoning.

Problem 8

In the Einstein model for a solid, atoms are treated as one-dimensional quantum mechanical oscillators that can each accept an arbitrary number of energy units above the ground state. Recall that the multiplicity $\Omega(N, q) = (q + N - 1)!/(q!(N - 1)!)$ gives the number of states available to a system containing N such oscillators, with q units of energy (of value ε) distributed among them, so that the total system energy $U = q\varepsilon$.

Hints: Two useful approximations for $\Omega(N, q)$ are the low-temperature limit ($q \ll N$), $\Omega(N, q) \sim (e\,N/q)^q$, and the high-temperature limit ($q \gg N$), $\Omega(N, q) \sim (e\,q/N)^N$, where e is the base of natural logarithms.

a) From the corresponding entropy $S(N, q)$, for each case (low and high temperature limit) determine the total system energy U in terms of temperature T and the energy unit ε.

b) Determine the heat capacity $C_V(T)$ of the solid and describe its limiting behavior as $T \to 0$ and $T \to \infty$. In the high-temperature limit, compare the classical result obtained by use of the equipartition theorem.

c) Explain the reason for the difference in behavior of the heat capacity in the high and low temperature limits.

Problem 9

Find the change in the internal energy of one mole of a monatomic ideal gas in a reversible isobaric expansion at 2.0 atm from a volume of $25\,\mathrm{m}^3$ to a volume of $30\,\mathrm{m}^3$.

Problem 10

It is well known that a clock placed in a gravitational potential $V(r)$ will run more slowly than one outside the influence of a gravitational field. This is described by General Relativity. The period P_0 of a clock measured by an observer outside the potential field, that is at a location $V = 0$, is related to the period P_r of the clock measured within the potential field by

$$P_0 = P_r(1 - V(r)/c^2).$$

a) What is the expression for the gravitational potential of the Earth near its surface as a function of height above the surface? Express the answer as a function of the gravitational acceleration, g, and the height, h.

b) If a laser is pointed vertically upwards, what is the fractional shift in frequency of a photon from the laser after it has travelled a distance of 20 meters from the Earth's surface?

c) What would be the fractional shift in frequency of a photon that travels from the surface of the Sun to a distance far away (where you can assume $V = 0$)?

Problem 11

In an experiment, electrons with a kinetic energy $K = 500$ MeV are scattered from a target of nuclei. Assume a nucleus has a uniform charge distribution in the shape of a sphere of radius r. The scattered electrons exhibit a diffraction pattern with minima separated by $\theta = 30°$.

a) Find the momentum of the incident electrons (in SI units).

b) Find the wavelength of the electrons.

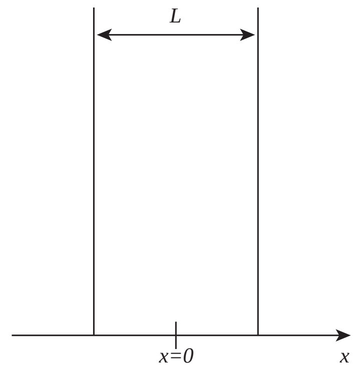
c) Estimate the charge distribution radius from the diffraction pattern.

d) Estimate the uncertainty in a proton's momentum in these nuclei.

Problem 12

An electron is trapped inside an infinite potential well of width $L = 10$ nm.

a) Show that the following wave-functions are solutions of the Schrödinger equation:



$$\Psi_n(x) = \sqrt{\frac{2}{L}}\cos(kx), \qquad k = n\pi/L, \qquad n = 1, 3, 5, ...$$

b) Sketch $\Psi_1(x)$ and $\Psi_3(x)$ into the box shown to the right.

c) The electron is in the ground state (the lowest possible energy state). Derive an expression for the electron's kinetic energy (the zero point energy) and calculate this energy in eV.

d) Write down the integral that gives the probability for finding the electron in the center half of the box (that is, between $x_1 = -2.5$ nm and $x_2 = +2.5$ nm). Write down the integral, but do not evaluate the expression.