The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are twelve equally weighted questions, each beginning on a new page. Read all twelve questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. If you need extra space for another problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic. **Calculators with stored equations or text are not allowed.** Dictionaries may be used if they have been approved by the proctor before the examination begins. **No other papers or books may be used.**

When you have finished, come to the front of the room and hand your examination paper to the proctor; first put all problems in numerical order and staple them together.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand your exam paper in on time, an appropriate number of points may be subtracted from your final score.
Constants

Electron charge \((e)\) 1.60 \times 10^{-19} \text{ C}

Electron rest mass \((m_e)\) 9.11 \times 10^{-31} \text{ kg (0.511 MeV/c}^2\)

Proton rest mass \((m_p)\) 1.673 \times 10^{-27} \text{ kg (938 MeV/c}^2\)

Neutron rest mass \((m_n)\) 1.675 \times 10^{-27} \text{ kg (940 MeV/c}^2\)

\(W^+\) rest mass \((m_{W^+})\) 80.4 \text{ GeV/c}^2

Planck’s constant \((h)\) 6.63 \times 10^{-34} \text{ J\cdot s}

Speed of light in vacuum \((c)\) 3.00 \times 10^8 \text{ m/s}

Boltzmann’s constant \((k_B)\) 1.38 \times 10^{-23} \text{ J/K}

Gravitational constant \((G)\) 6.67 \times 10^{-11} \text{ N\cdot m}^2/\text{kg}^2

Permeability of free space \((\mu_0)\) 4\pi \times 10^{-7} \text{ H/m}

Permittivity of free space \((\varepsilon_0)\) 8.85 \times 10^{-12} \text{ F/m}

Mass of Earth \((M_{\text{Earth}})\) 5.98 \times 10^{24} \text{ kg}

Mass of Moon \((M_{\text{Moon}})\) 7.35 \times 10^{22} \text{ kg}

Radius of Earth \((R_{\text{Earth}})\) 6.38 \times 10^6 \text{ m}

Radius of Moon \((R_{\text{Moon}})\) 1.74 \times 10^6 \text{ m}

Radius of Sun \((R_{\text{Sun}})\) 6.96 \times 10^8 \text{ m}

Earth - Sun distance \((R_{\text{ES}})\) 1.50 \times 10^{11} \text{ m}

Density of iron at low temperature \((\rho_{Fe})\) 7.88 \times 10^3 \text{ kg/m}^3

Classical electron radius \((r_0)\) 2.82 \times 10^{-15} \text{ m}

Gravitational acceleration on Earth \((g)\) 9.8 \text{ m/s}^2

Atomic mass unit 1.66 \times 10^{-27} \text{ kg}

Specific heat of oxygen \((c_V)\) 21.1 \text{ J/mole\cdot K}

Specific heat of oxygen \((c_P)\) 29.4 \text{ J/mole\cdot K}

Moments of Inertia

For a hoop of mass \(M\) and radius \(R\), about its symmetry axis: \(MR^2\).

For a disk or cylinder of mass \(M\) and radius \(R\), about its symmetry axis: \((1/2)MR^2\).

For a solid sphere of mass \(M\) and radius \(R\), about any symmetry axis: \((2/5)MR^2\).

For a spherical shell of mass \(M\) and radius \(R\), about any symmetry axis: \((2/3)MR^2\).
Problem 1

A particle of mass $m$ is subject to a position-dependent force with potential energy given by

$$V(x) = Ax^2 e^{-b^2 x^2}$$

where $A$ and $b$ are positive constants. For simplicity assume the motion is strictly confined to the $x$-axis.

a) What is the force as a function of position $x$?

b) What are the equilibrium positions? Indicate whether these positions are stable or unstable.

c) What is the upper limit to the total energy a particle can have and still be bound to remain near the origin?

d) What is the period of small oscillations around each point of stable equilibrium?
Problem 2

A ball of mass $m$ is projected with speed $v$ into the barrel of a spring gun of mass $M$ as shown below. The spring gun is initially at rest on a frictionless surface. A small latch locks the ball in the barrel at the point of maximum compression of the spring. Assume no energy is lost to friction.

a) What is the speed of the gun after the ball comes to rest inside the barrel?

b) What fraction of the initial kinetic energy is stored in the spring?

c) If the latch is then released, what are the final velocities of the ball and gun?
Problem 3

A billiard ball of mass $m$ and radius $b$ sitting on a table is struck at the center imparting an initial linear velocity $v_0$ but zero angular velocity. The coefficient of sliding friction between the ball and the table is $\mu$.

![Diagram of a billiard ball being struck on a table]

a) At what velocity $v$ does the ball begin to roll without slipping?

b) How far does the ball travel before it begins to roll without slipping?
Problem 4

The device in the figure below consists of two concentric, conducting cylinders. The inner cylinder, of radius $a$, is at nominal ground but carries a current $I_0$ in the $+z$ direction. The outer cylinder, of radius $b$, is at an electrical potential $-V_0$ and carries no current.

![Diagram of concentric cylinders with currents and potentials](image)

a) Ignoring fringing fields, what is the electric field between the cylinders?

b) Ignoring fringing fields, what is the magnetic field between the cylinders?

c) Derive a condition on $I_0$ and $V_0$ that would allow a positive ion of charge $q$ moving in the $+z$ direction with velocity $v_0$ to continue moving without deflection between the cylinders.

d) With protons entering the region between the cylinders with kinetic energy of 1 keV, $V_0 = 1$ kV, and $b/a = 10$, what current would be required to maintain a linear trajectory?
Problem 5

An infinite solenoid with $N$ coils per unit length and a cross-sectional area $\pi a^2$ carries a current $I$. A circular plastic hoop of radius $b > a$ surrounds the solenoid oriented such that the plane of the hoop is perpendicular to the solenoid axis. A particle with charge $q$ and mass $m$, initially at rest, is attached to the hoop but is able to slide freely along the hoop. If the current to the solenoid is turned off slowly, find the velocity of the particle after the current has reached zero. Ignore the effects of radiation or gravity.
Problem 6

A parallel-plate capacitor with plate area $A$, width $h$, and plate separation $d$ is charged with a battery to a potential $V_0$. The battery is disconnected and a dielectric slab of thickness $d$, mass $m$, and dielectric constant $\kappa$ is slid between the plates such that the volume between the capacitor plates is completely filled with the dielectric. Assuming friction is negligible, calculate the work done on the dielectric slab as it is inserted.
Problem 7

An ideal gas in a container with a piston initially has volume $V_a$ and temperature $T_a$. The piston is allowed to move quasi-statically so that the gas volume increases to $V_b$. Heat flow from an “auxiliary system” is controlled so the $T,V$ curve follows a straight line:

$$T(V) = T_a - \alpha (V - V_a),$$

where $-\alpha$ is the slope of the line shown in the figure ($\alpha > 0$).

Recall that the heat capacity of an ideal gas is $C_v = \frac{3}{2}nR$.

Find expressions for:

a) the total work done on the gas,

b) the total heat transferred to the gas,

c) the total energy change of the gas,

d) the total entropy change of the gas. (This should be negative.)

e) Given only the fact that the entropy change of the gas is negative, what can you say about the entropy change of the “auxiliary system”? 
Problem 8

30 grams of ice at 0°C are placed into a cup containing 100 grams of water at 20°C. The cup has perfectly insulating walls of negligible heat capacity.

a) What is the final temperature of the mixture after it reaches thermal equilibrium?

b) How much ice, if any, remains?

c) What is the total entropy change of the ice plus water mixture?

Recall that the specific heat per gram of water is 4.18 J/°C and that it takes 333 J to convert 1 gram of ice into water at 0°C.
Problem 9

A small, inert particle of mass \( m = 1.0 \times 10^{-9} \) kg is immersed in a liquid at temperature \( T = 300 \) K. Let earth’s gravitational field act in the \( z \) direction. Determine the RMS velocity of the particle in the \( x \) (or \( y \)) direction.
Problem 10

A neutral pion (rest mass = 135 MeV), moving with a kinetic energy of 1 GeV, decays into two photons.

a) What are the minimum and maximum possible photon energies?

b) At what decay angle $\theta$ with respect to the original pion flight direction will the photons be emitted when they have equal energy?
Problem 11

The radial wave function for a hydrogen atom for states with \( l = n - 1 \) has the form

\[
R(r) = Ar^{n-1}e^{-\frac{r}{na_0}}
\]

where \( n \) is the radial quantum number, \( a_0 \) is the Bohr radius, and \( A \) is a normalization factor.

a) Find an expression for the probability of finding the electron at radius \( r \) and within a small interval \( \Delta r \) in terms of \( r, \Delta r, n, \) and \( a_0 \).

b) Find the expectation values of \( r \) as a function of \( n \) and \( a_0 \).

Possibly useful: \( \int_0^\infty x^n e^{-x} \, dx = n! \)
Problem 12

The apparatus depicted below was used in 1914 to study the collisions of electrons with atoms. The chamber is filled with mercury vapor at low pressure. Electrons are emitted thermally at the heated cathode on the left. A variable voltage $V$ is applied between the cathode and a slotted anode. Another voltage of opposite polarity $V_r$ is applied between the anode and an electron-collecting plate. The current $I$ between the cathode and the collecting plate is measured by an electrometer.

![Diagram of the apparatus](image)

**a)** What is the purpose of applying the voltage $V$ between the cathode and the anode? Why is the anode slotted? What is the purpose of the voltage $V_r$ that is applied between the anode and the collecting plate?

**b)** A typical plot the the current $I$ versus the voltage $V$ for this experiment is shown below. What is the significance of the peaks in the plot? Why are they regularly spaced?