

Exam #: \_\_\_\_\_

Printed Name: \_\_\_\_\_

Signature: \_\_\_\_\_

PHYSICS DEPARTMENT  
UNIVERSITY OF OREGON

Master's Final Examination and Ph.D. Qualifying Examination

PART II

Wednesday, September 30, 2009, 9:00 a.m. to 1:00 p.m.

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

There are eight equally weighted questions, each beginning on a new page. Read all eight questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. When you start a new problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic and will be provided. **Personal calculators are not allowed.** Dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries are not allowed.** **No other papers or books may be used.**

When you have finished, come to the front of the room, put all problems in numerical order and staple them together with this sheet on top. Then hand your examination paper to the proctor.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand your exam paper in on time, an appropriate number of points may be subtracted from your final score.

## Constants

Electron charge ( $e$ )	$1.60 \times 10^{-19} \text{ C}$
Electron rest mass ( $m_e$ )	$9.11 \times 10^{-31} \text{ kg}$ (0.511 MeV/ $c^2$ )
Proton rest mass ( $m_p$ )	$1.673 \times 10^{-27} \text{ kg}$ (938 MeV/ $c^2$ )
Neutron rest mass ( $m_n$ )	$1.675 \times 10^{-27} \text{ kg}$ (940 MeV/ $c^2$ )
Atomic mass unit (AMU)	$1.7 \times 10^{-27} \text{ kg}$
Atomic weight of a nitrogen atom	14 AMU
Atomic weight of an oxygen atom	16 AMU
Planck's constant ( $h$ )	$6.63 \times 10^{-34} \text{ J}\cdot\text{s}$
Speed of light in vacuum ( $c$ )	$3.00 \times 10^8 \text{ m/s}$
Boltzmann's constant ( $k_B$ )	$1.38 \times 10^{-23} \text{ J/K}$
Gravitational constant ( $G$ )	$6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$
Permeability of free space ( $\mu_0$ )	$4\pi \times 10^{-7} \text{ H/m}$
Permittivity of free space ( $\epsilon_0$ )	$8.85 \times 10^{-12} \text{ F/m}$
Mass of earth ( $M_E$ )	$5.98 \times 10^{24} \text{ kg}$
Equatorial radius of earth ( $R_E$ )	$6.38 \times 10^6 \text{ m}$
Radius of sun ( $R_S$ )	$6.96 \times 10^8 \text{ m}$
Classical electron radius ( $r_0$ )	$2.82 \times 10^{-15} \text{ m}$
Specific heat of oxygen ( $c_V$ )	21.1 J/mole $\cdot$ K
Specific heat of oxygen ( $c_P$ )	29.4 J/mole $\cdot$ K
Specific heat of water ( $0^\circ \text{ C} < T < 100^\circ \text{ C}$ )	4.18 J/(g $\cdot$ K)
Latent heat, ice $\rightarrow$ water	334 J/g
Latent heat, water $\rightarrow$ steam	2257 J/g
Gravitational acceleration on Earth ( $g$ )	9.8 m/s <sup>2</sup>
1 atmosphere	$1.01 \times 10^5 \text{ Pa}$

## Pauli spin matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (1)$$

**Hydrogen atom energy levels** for a potential energy  $V(r) = e^2/r$ .

$$E_n = -\frac{me^4}{2\hbar^2} \frac{1}{n^2}. \quad (2)$$

## Integrals

$$\int_{-\infty}^{\infty} dx x^{2n} e^{-ax^2} = \frac{1 \times 3 \times 5 \times \dots \times (2n-1)}{2^n a^n} \sqrt{\frac{\pi}{a}}$$

$$\int_0^{\infty} dx e^{-cx} x^n = \frac{n!}{c^{n+1}}$$

## Problem 1

A non-relativistic particle with mass  $m$  in three dimensional space, with coordinates  $(x, y, z)$ , is subject to the potential

$$V = \frac{m\omega^2}{2} [x^2 + y^2 + z^2] .$$

Here  $\omega$  is a parameter that describes the strength of the potential.

- (a) Choose a convenient scheme for labeling the energy eigenstates of this system. Using your scheme, write the energy eigenvalues as a function of the quantum numbers of the states. If you are familiar with the solutions of the Schrödinger equation with this potential, you can simply state the result.
- (b) Give the energies of the ten states with the lowest energies, noting which states have energies that are degenerate with the energies of other states.
- (c) Suppose that we now change the potential to

$$V = \frac{m\omega^2}{2} [x^2 + y^2 + (1 + \delta)^2 z^2] .$$

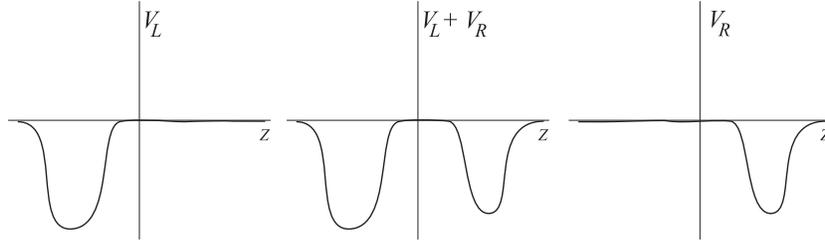
Here  $\delta$  a dimensionless parameter that is not too large, say  $\delta < 0.1$ . Again, give the energies of the ten states with the lowest energies, noting which states have energies that are degenerate with the energies of other states.

## Problem 2

Consider a quantum mechanical particle in a double-well potential,

$$H = \frac{\vec{p}^2}{2m} + V_L(\vec{x}) + V_R(\vec{x}) .$$

The two potential wells, represented by  $V_L(\vec{x})$  and  $V_R(\vec{x})$ , are similar but do not necessarily have exactly the same shape.



If the two potential wells are deep and well separated, the wave functions for the lowest two energy levels can be approximated by solutions  $|L\rangle$  and  $|R\rangle$  of

$$\left[ \frac{\vec{p}^2}{2m} + V_L(\vec{x}) \right] |L\rangle = E_{L,0} |L\rangle , \quad \left[ \frac{\vec{p}^2}{2m} + V_R(\vec{x}) \right] |R\rangle = E_{R,0} |R\rangle .$$

For simplicity, we will make a two-state approximation for the particle, restricting our Hilbert space to the the space spanned by these two states.

Now suppose that we account for the possibility of tunneling between the two wells, still considering only the two states  $|L\rangle$  and  $|R\rangle$ . Define  $E_L$ ,  $E_R$  and  $\alpha$  by  $E_L = \langle L|H|L\rangle$ ,  $E_R = \langle R|H|R\rangle$  and  $\alpha = \langle L|H|R\rangle$ . Since the barrier between the two wells is not infinite,  $\alpha \neq 0$ .

- Express the problem of finding the energy eigenvalues for this problem as a simple problem involving a  $2 \times 2$  matrix.
- Now take the double well to be symmetric, so that  $E_L = E_R$ . Let the time dependent state for the system be

$$|\psi(t)\rangle = c_L(t)|L\rangle + c_R(t)|R\rangle .$$

Let the initial condition be  $|\psi(t)\rangle = |L\rangle$ . Find  $c_L(t)$  and  $c_R(t)$  for later times. Show that after a time  $T$  the particle returns to the  $|L\rangle$  state except possibly for a phase factor,  $|\psi(T)\rangle = e^{i\phi}|L\rangle$ . Find  $T$  in terms of  $E_L$ ,  $E_R$  and  $\alpha$ . If there is more than one solution for  $T$ , find the smallest one.

- What are the energy eigenstates and eigenvalues for the hamiltonian of part (b)? (You need not derive the eigenstates and eigenvalues if you know what they are, but you should give precise answers and show that they are correct.) How are the energy eigenvalues related to the time  $T$  that you found in part (b)?

### Problem 3

This problem concerns the spin degree of freedom for a spin 1/2 particle, with  $|\uparrow\rangle$  representing spin up along the  $z$  axis and  $|\downarrow\rangle$  representing spin down along the  $z$ -axis.

- (a) Consider a single spin 1/2 particle in the state  $|\uparrow\rangle$ . If we rotate it through an angle  $\theta = +\pi$  around the  $y$ -axis, we get a state of the form  $e^{i\phi}|\downarrow\rangle$ . Calculate the phase factor  $e^{i\phi}$ . What is the result if we rotate the state  $|\downarrow\rangle$  through an angle  $\theta = \pi$  around the  $y$ -axis?
- (b) Now suppose that we have two particle spins. Both particles have spin 1/2. Let  $R(\hat{y}, \pi)$  be the operator that rotates the spins about the  $y$ -axis through an angle  $\pi$ . (That is,  $R(\hat{y}, \pi)$  rotates both spins through the same angle about the same axis.) Apply the operator  $R(\hat{y}, \pi)$  to the state

$$\frac{1}{\sqrt{2}} [|\uparrow\rangle|\downarrow\rangle + |\downarrow\rangle|\uparrow\rangle] .$$

What do you get? Now apply the operator  $R(\hat{y}, \pi)$  to the state

$$\frac{1}{\sqrt{2}} [|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle] .$$

What do you get?

- (c) Using your knowledge of angular momentum and the rotation group, what is the result of applying *any* rotation operator  $R(\vec{n}, \theta)$  to the state

$$\frac{1}{\sqrt{2}} [|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle] .$$

You need not give a derivation of your result. Is your result in part (b) consistent with your general statement here?

## Problem 4

Consider an electron subject to the hamiltonian

$$H = \frac{\mathbf{p}^2}{2m} - \frac{e^2}{r} - \frac{e}{2mc} (\mathbf{L} + 2\mathbf{S}) \cdot \mathbf{B} . \quad (3)$$

This represents a hydrogen atom (with no spin-orbit coupling) in a magnetic field. The magnetic field  $\mathbf{B}$  is uniform in space and time independent. The vectors  $\mathbf{L}$  and  $\mathbf{S}$  represent the orbital angular momentum and electron spin, respectively.

- (a) If the magnetic field is turned off, what is the ground state energy,  $E_1^{(0)}$ ? What is the next lowest energy level,  $E_2^{(0)}$ ? (You are not expected to derive these. Simply write them down.)
- (b) How many quantum states correspond to each of these levels?
- (c) Now suppose that the external magnetic field is applied. Consider just what happens to the two energy levels from part (a), which are now split. What are the corresponding new energy levels? How many distinct new energy levels are there? For each of them, how many quantum states correspond to each energy level? Please explain how you label the states and draw a diagram showing the new levels and which states correspond to which levels.

### Problem 5

Model the Earth's atmosphere as an ideal gas in thermal equilibrium at a *uniform temperature* of 304 K, subject to the Earth's gravitational acceleration  $g$ , which you may take to be uniform.

- (a) Find the ratio of the density of diatomic oxygen ( $O_2$ ) molecules at sea level to that at the summit of Mount Everest, which is 8.8 km above sea level.
- (b) The ratio of the mass density of diatomic nitrogen ( $N_2$ ) molecules to that of diatomic oxygen is 3 at sea level. Compute the ratio of the mass density of diatomic nitrogen to that of diatomic oxygen at the summit of Mount Everest, subject to the earlier assumptions.

## Problem 6

A rigid, stationary diatomic molecule has one spin  $\frac{1}{2}$  on each atom. Each spin can be either up or down. When the spins are parallel, the energy of the molecule is  $J$ , while when they are anti-parallel, its energy is  $-J$ .

- (a) Find the partition function for this system if it is in equilibrium at a temperature  $T$ .
- (b) Calculate the free energy, entropy, and specific heat of this system. Sketch a plot of the specific heat as a function of temperature, and indicate roughly the position of any features (e.g., the maximum, if any) in this plot.

## Problem 7

In the process of pumping up a bicycle tire, a liter of air at atmospheric pressure is compressed adiabatically to a pressure of 7 atmospheres. (Air is mostly diatomic nitrogen and oxygen).

- (a) What is the final volume of this air after compression?
- (b) How much work is done in compressing the air?
- (c) If the temperature of the air was initially (i.e., before the compression) 300 K, what is the temperature after compression?

### Problem 8

A large number  $N \gg 1$  of electrons move at absolute zero temperature in a macroscopic box of volume  $V$ . You may ignore the interactions between the electrons.

- (a) Determine the energy of the most energetic electron in this system.
- (b) What is the total energy of the electron gas?
- (c) Determine the pressure exerted by this gas.