Exam #: ______________________

Printed Name: ______________________

Signature: ______________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Unified Graduate Examination

Part IV

Statistical Mechanics

Friday, September 22, 2017, 13:30 to 16:10

The examination booklet is numbered in the upper right-hand corner of the cover page. Print and then sign your name in the spaces provided on the cover page. For identification purposes, be sure to submit this page together with your answers when the exam is finished.

There are four questions, each beginning on a new page. Read all four questions before attempting any answer. You may answer as many questions as you wish, however, only your top three scores will be used in the evaluation of your performance for a Ph.D. pass in this area, or your top two scores for a master's pass.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. When you start a new problem, start a new page. Place both the exam number and the question number on all pages you wish to have graded. You are encouraged to use the constants on the following page, where appropriate, to help you solve the problems.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic and will be provided. **Personal calculators are not allowed**. Dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries are not allowed. No other papers or books may be used.**

When you have finished, come to the front of the room. For each problem, put the pages in order and paper clip them together. Then put all problems in numerical order and place them in the envelope provided. Finally, hand the envelope to the proctor.

Constants

Electron charge (e)	$1.60 \times 10^{-19}\,\mathrm{C}$
Electron rest mass (m_e)	$9.11 \times 10^{-31}\,\mathrm{kg}$ ($0.511\,\mathrm{MeV/c^2}$)
Proton rest mass (m_p)	$1.673 \times 10^{-27}\,\mathrm{kg}$ ($938\,\mathrm{MeV/c^2}$)
Neutron rest mass (m_n)	$1.675 \times 10^{-27}\,\mathrm{kg}$ ($940\,\mathrm{MeV/c^2}$)
Atomic mass unit (AMU)	$1.66 \times 10^{-27}\,\mathrm{kg}$
Atomic weight of a hydrogen atom	1 AMU
Atomic weight of a nitrogen atom	14 AMU
Atomic weight of an oxygen atom	16 AMU
Planck's constant (h)	$6.63 \times 10^{-34}\,\mathrm{J{\cdot}s}$
Bohr Magneton (μ_B)	$9.27 \times 10^{-28}\,\mathrm{J/G}$
Speed of light in vacuum (c)	$3.00 \times 10^{8}\,\mathrm{m/s}$
Boltzmann's constant (k_B)	$1.38 \times 10^{-23}\,\mathrm{J/K}$
Gravitational constant (G)	$6.67 \times 10^{-11}\mathrm{N{\cdot}m^2/kg^2}$
Permeability of free space (μ_0)	$4\pi \times 10^{-7}\,\mathrm{H/m}$
Permittivity of free space (ϵ_0)	$8.85 \times 10^{-12}\,\mathrm{F/m}$
Mass of earth (M_E)	$5.98 \times 10^{24}\,\mathrm{kg}$
Equatorial radius of earth (R_E)	$6.38 \times 10^{6}\,\mathrm{m}$
Mass of Sun (M_S)	$1.99 \times 10^{30}\,\mathrm{kg}$
Radius of Sun (R_S)	$6.96 \times 10^{8}\,\mathrm{m}$
Classical electron radius (r_0)	$2.82 \times 10^{-15}\,\mathrm{m}$
Density of water	1.0 kg/liter
Density of ice	0.917 kg/liter
Specific heat of water	4184 J/(kg K)
Specific heat of ice	2060 J/(kg K)
Latent heat of fusion of water	334.2 kJ/kg
Heat of vaporization of water	2260 kJ/kg
Specific heat of oxygen (c_V)	$21.1\,\mathrm{J/mole{\cdot}K}$
Specific heat of oxygen (c_P)	$29.4\,\mathrm{J/mole{\cdot}K}$
Gravitational acceleration on Earth (g)	$9.8\,\mathrm{m/s^2}$
1 atmosphere	1.01×10^{5} Pa

Problem 4.1

You store 10.0 kg of ice at -20.0 deg C in a perfectly thermally insulating vessel. You then open it, quickly pour 2.0 kg of water at +40.0 deg C on the ice, and immediately seal the vessel.

(a) [5 points] What will the temperature be when the system has reached thermal equilibrium? [Ignore thermal effects of air.]

(b) [5 points] How much ice (in kg) will there be in the end?

Problem 4.2

A very tall vertical cylinder contains $N \gg 1$ non-interacting indistinguishable massive monoatomic particles. It sits in a uniform gravitational field g directed downwards (i.e., along the axis of the cylinder). Throughout this problem, treat this system classically and non-relativistically.

(a) [5 points] Calculate the Helmholtz free energy of this system, approximating the cylinder as infinite.

(b) [2 points] Calculate the entropy of this system, approximating the cylinder as infinite.

(c) [3 points] Imagine that we can now tune the acceleration due to gravity. Suppose we start the system at a temperature T_0 and gravitational acceleration g_0, and then slowly increase the gravitational acceleration to a larger value $g_F > g_0$, while not allowing the particles inside the cylinder to exchange heat with their surroundings, or to escape the cylinder. Calculate the final temperature T_F.

Problem 4.3

Inside a box of volume V there are N ideal gas particles held at a fixed temperature T. Each particle has a mass $2m$ and can, with an energy cost ϵ, dissociate into two separate ideal gas particles each of mass m. Given the thermodynamic state of the system a fraction $0 < f < 1$ of the particles will be dissociated. Assume that the particles have no internal degrees of freedom, and recall that the intrinsic chemical potential of an ideal gas with particle mass m is

$$\mu = k_B T \ln \left[\frac{N}{V} \left(\frac{h^2}{2\pi m k_B T} \right)^{3/2} \right]. \tag{1}$$

(a) (1 point) What is the pressure inside the box as a function of f and the thermodynamic variables?

(b) (2 points) What is the internal energy as a function of f and the thermodynamic variables?

(c) (4 points) What is the fraction f of ionized gas particles as a function of the thermodynamic variables?

(d) (3 points) What is the form of $P(T)$ as $T \to 0$ and as $T \to \infty$? Estimate the temperature at which you expect to P to transition between these asymptotes. What fraction of molecules are dissociated at those temperatures?

4.4

A circular cylinder of radius R turns around its axis at constant angular frequency ω. The cylinder contains a classical ideal gas with atomic mass M at temperature T.

(a) (2 points) Assume the system has come to equilibrium in the rotating frame of the cylinder, so that the atoms have zero *average velocity* in that frame. State the centrifugal force and the corresponding potential energy as a function of distance r from the axis.

(b) (3 points) Write down the classical canonical Boltzmann probability density for a single gas particle as a function of its phase-space coordinates, in the rotating frame. You're allowed to ignore the Coriolis force and include only the effect of the centrifugal force from part **(a)**. In your answer, identify and define the canonical partition function Z. You need not obtain a closed-form expression for Z.

(c) (3 points) Using part **(b)**, find an expression for the density $n(r)$ of the classical ideal gas as a function of the radial distance $r < R$ from the axis, in terms of $n(0)$ on the axis. Here, $n(r)$ is defined to be the volume number density in units of meters^{-3}.

(d) (2 points) Now let's see what happens if we include the Coriolis force,

$$\boldsymbol{F}_{\mathrm{C}} = 2M\omega\dot{\boldsymbol{r}} \times \hat{\boldsymbol{z}}$$

where $\hat{\boldsymbol{z}}$ is the unit vector along the axis. This happens to be the same as the Lorentz force on a particle of charge e in a uniform magnetic field

$$\boldsymbol{B} \equiv \frac{2M\omega}{e}\hat{\boldsymbol{z}}.$$

Will the result for $n(r)$ be different from **(c)** if you include a vector potential that corresponds to this magnetic field in the Hamiltonian of a single point charge e with mass M? Explain your reasoning.