Exam #: ________________________

Printed Name: ________________________

Signature: ________________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Unified Graduate Examination

Part IV

Statistical Mechanics and Thermodynamics

Tuesday, September 29, 2015, 13:30 to 16:10

The examination booklet is numbered in the upper right-hand corner of the cover page. Print and then sign your name in the spaces provided on the cover page. For identification purposes, be sure to submit this page together with your answers when the exam is finished.

There are four questions, each beginning on a new page. Read all four questions before attempting any answer. You may answer as many questions as you wish, however, only your top three scores will be used in the evaluation of your performance for a Ph.D. pass in this area, or your top two scores for a master's pass. (These scores will be added to your aggregate if taking the exam "under the old rules".)

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. When you start a new problem, start a new page. Place both the exam number and the question number on all pages you wish to have graded. You are encouraged to use the constants on the following page, where appropriate, to help you solve the problems.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic and will be provided. **Personal calculators are not allowed**. Dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries are not allowed. No other papers or books may be used.**

When you have finished, come to the front of the room. For each problem, put the pages in order and staple them together. Then put all problems in numerical order and place them in the envelope provided. Finally, hand the envelope to the proctor.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg (0.511 MeV/c^2)
Proton rest mass (m_p)	1.673×10^{-27} kg (938 MeV/c^2)
Neutron rest mass (m_n)	1.675×10^{-27} kg (940 MeV/c^2)
Atomic mass unit (AMU)	1.66×10^{-27} kg
Atomic weight of a hydrogen atom	1 AMU
Atomic weight of a nitrogen atom	14 AMU
Atomic weight of an oxygen atom	16 AMU
Planck's constant (h)	6.63×10^{-34} J·s
Bohr Magneton (μ_B)	9.27×10^{-24} J/T
Speed of light in vacuum (c)	3.00×10^{8} m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Gravitational constant (G)	6.67×10^{-11}N·m^2/kg^2
Permeability of free space (μ_0)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space (ϵ_0)	8.85×10^{-12} F/m
Mass of earth (M_E)	5.98×10^{24} kg
Equatorial radius of earth (R_E)	6.38×10^{6} m
Mass of Sun (M_S)	1.99×10^{30} kg
Radius of Sun (R_S)	6.96×10^{8} m
Classical electron radius (r_0)	2.82×10^{-15} m
Density of water	1.0 kg/liter
Density of ice	0.917 kg/liter
Specific heat of water	4180 J/(kg K)
Specific heat of ice	2050 J/(kg K)
Heat of fusion of water	334 kJ/kg
Heat of vaporization of water	2260 kJ/kg
Specific heat of oxygen (c_V)	21.1 J/mole·K
Specific heat of oxygen (c_P)	29.4 J/mole·K
Gravitational acceleration on Earth (g)	9.8 m/s^2
1 atmosphere	1.01×10^{5} Pa

Problem 1

A star of radius R_s with surface temperature T_s radiates light according to the blackbody spectrum into the cold background of space ($T \sim 0$ K). At a distance d away from the star, a planet with radius R_p ($d \gg R_p$) absorbs this radiation. Various processes on the planet's surface keep it at a uniform temperature T_p. Ignore any reflective radiation between these two bodies.

a. (2 points) What is the equilibrium temperature of the planet assuming it is a perfect blackbody? How does the equilibrium temperature of the planet change with the size of the planet (for $d \gg R_p$)? Ignore effects of an atmosphere.

b. (2 points) What fraction of the star's original power is radiated back to it by the planet? Given that $d \gg R_s$ and $d \gg R_p$, to first order, should you worry about this effect?

c. (3 points) In an effort to cool down the planet for habitation, a very capable alien civilization builds a thin concentric shell of radius R_l around the star that acts as a perfect blackbody. If the surface temperature of the star remains constant, what is the equilibrium temperature of the shell T_l? What is the new equilibrium temperature of the planet $T_p^{'}$? As a fraction of the initial equilibrium temperature T_p, what is the maximum temperature shift the aliens can create on the planet using a shell with $d \geq R_l \geq R_s$?

d. (3 points) Given the same situation as in Part c, except that the net power output of the star remains constant, what is the new equililbrium temperature of the planet $T_p^{'}$?

Problem 2

An ideal gas of N identical particles of mass m and fixed temperature T is confined to a vertical cylinder of radius R which has a sealed base and infinite height in a uniform gravitational field g.

a. (2 points) What is the natural length scale, λ, along which the pressure in the cylinder decreases from the initial height $h = 0$?

b. (4 points) What is the pressure as a function of the height h in the cylinder?

c. (2 points) What is the internal energy of the gas?

d. (2 points) What is the specific heat of the gas? Explain in words why it is higher than the specific heat of an ideal gas that is not in a gravitational field.

Problem 3

A surface has $B \gg 1$ sites onto which molecules of a gas in contact with the surface can bind. The gas is held at a fixed temperature and pressure. Each site can either be occupied, at energy $-\epsilon$ (with $\epsilon > 0$), or unoccupied at zero energy.

a. (2 points) For any non-zero temperature, not all sites will be occupied, why? What will happen to the fraction of occupied sites at $\epsilon \to \infty$? Justify your answers in words.

b. (2 points) What is the entropy associated with having N particles bound to the surface? Simplify S by assuming that N, B, and $B - N$ are all $\gg 1$. For what value of $f = N/B$ is the surface entropy maximized? When appropriate, you may use Stirling's approximation,

$$\ln n! \approx n \ln n - n. \tag{1}$$

c. (3 points) What is the energy of the surface when $N \gg 1$ particles are bound to it? What is the chemical potential μ for particles on the surface? Express your answers in terms of: B, ϵ, and the fraction f of occupied sites.

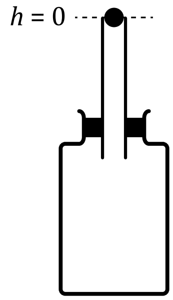
d. (3 points) If the adsorbed particles are in thermal equilibrium with the particles in the ideal gas, what is f? Express your answer in terms of ϵ, T, the number density of particles in the ideal gas n, and fundamental constants.

Hint: The chemical potential of an ideal gas is

$$\mu = k_B T \ln \left(n \left(\frac{2\pi\hbar^2}{m k_B T} \right)^{3/2} \right) \tag{2}$$

Problem 4

A large jar of volume V_o contains N ideal gas particles at a temperature T_o. Fitted to the jar is a long, cylindrical tube of cross-sectional area A (its volume is included in V_o). Initially, the pressure inside and outside the jar is p_o and the gas is homogeneous. A ball of mass m, which forms an air-tight and frictionless seal with the tube, is dropped into the tube.



a. (2 points) If the system reaches isothermal equilibrium, what is the equilibrium height $h_{eq}(< 0)$ of the ball? Assume the gas remains isothermal with temperature $T_\circ$ as it evolves. What is the pressure inside the jar? How does this equilibrium height depend on the temperature T_o?

b. (2 points) If the system reaches adiabatic equilibrium, what is the equilibrium height $h_{eq}(< 0)$ of the ball? Assume the gas remains isothermal as it evolves but that its temperature changes adiabatically. What is the pressure inside the jar?

c. (2 points) After the system reaches adiabatic equilibrium in Part b, what is the temperature T of the gas inside the jar? You may leave your answer in terms of h_{eq} from Part b.

d. (4 points) Assuming the system evolves adiabatically, find an expression for the frequency f of small oscillations of the ball about its equilibrium position in Part b.