Exam #: ________________________

Printed Name: ________________________

Signature: ________________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Unified Graduate Examination

Part III

Quantum Mechanics

Wednesday, June 21, 2017, 10:00 to 12:40

The examination booklet is numbered in the upper right-hand corner of the cover page. Print and then sign your name in the spaces provided on the cover page. For identification purposes, be sure to submit this page together with your answers when the exam is finished.

There are four questions, each beginning on a new page. Read all four questions before attempting any answer. You may answer as many questions as you wish, however, only your top three scores will be used in the evaluation of your performance for a Ph.D. pass in this area, or your top two scores for a master's pass.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. When you start a new problem, start a new page. Place both the exam number and the question number on all pages you wish to have graded. You are encouraged to use the constants on the following page, where appropriate, to help you solve the problems.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic and will be provided. **Personal calculators are not allowed**. Dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries are not allowed. No other papers or books may be used.**

When you have finished, come to the front of the room. For each problem, put the pages in order and paper clip them together. Then put all problems in numerical order and place them in the envelope provided. Finally, hand the envelope to the proctor.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg (0.511 MeV/c^2)
Proton rest mass (m_p)	1.673×10^{-27} kg (938 MeV/c^2)
Neutron rest mass (m_n)	1.675×10^{-27} kg (940 MeV/c^2)
Atomic mass unit (AMU)	1.66×10^{-27} kg
Atomic weight of a hydrogen atom	1 AMU
Atomic weight of a nitrogen atom	14 AMU
Atomic weight of an oxygen atom	16 AMU
Planck's constant (h)	6.63×10^{-34} J·s
Bohr Magneton (μ_B)	9.27×10^{-28} J/G
Speed of light in vacuum (c)	3.00×10^{8} m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Gravitational constant (G)	6.67×10^{-11} N·m^2/kg^2
Permeability of free space (μ_0)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space (ϵ_0)	8.85×10^{-12} F/m
Mass of earth (M_E)	5.98×10^{24} kg
Equatorial radius of earth (R_E)	6.38×10^{6} m
Mass of Sun (M_S)	1.99×10^{30} kg
Radius of Sun (R_S)	6.96×10^{8} m
Classical electron radius (r_0)	2.82×10^{-15} m
Density of water	1.0 kg/liter
Density of ice	0.917 kg/liter
Specific heat of water	4180 J/(kg K)
Specific heat of ice	2050 J/(kg K)
Heat of fusion of water	334 kJ/kg
Heat of vaporization of water	2260 kJ/kg
Specific heat of oxygen (c_V)	21.1 J/mole·K
Specific heat of oxygen (c_P)	29.4 J/mole·K
Gravitational acceleration on Earth (g)	9.8 m/s^2
1 atmosphere	1.01×10^{5} Pa

Problem 3.1

(a) [5 points] Consider a spinless particle with mass m in the three-dimensional potential $V(r) = Cr^2$ with $C > 0$ some constant. What are the energy eigenvalues? What are the degeneracies of the three lowest energy eigenvalues?

(b) [5 points] Suppose instead that five identical noninteracting particles with mass m move in this potential. What is the ground state energy of this system (at zero temperature) if the particles have (i) spin 1/2, (ii) spin 1, (iii) spin 3/2?

3.2

Estimate the ground state energy of a hydrogen atom using the uncertainty principle as follows:

Suppose the electron is localized to a region of size of order r around the proton.

a) What is the order of magnitude of the momentum of the electron? (1 point) What is the order of magnitude of its kinetic energy, assuming it is moving at speeds $\ll c$, the speed of light? (1 point) What is the order of magnitude of its potential energy? (1 point) Express your answer in terms of r and fundamental constants.
By minimizing your expression for the total energy (sum of potential plus kinetic), estimate the order of magnitude of the ground state energy E_0. (2 points)

b) Now repeat the above calculation for a single electron orbiting a nucleus containing Z protons, and do *not* assume that the speed of the electron is $\ll c$. Show that the ground state energy goes to $-\infty$ at some critical value of Z, and calculate that critical value. (5 points)

Problem 3.3

Consider the set of coherent states for a simple harmonic oscillator

$$|z\rangle \;=\; A\sum_{n=0}^{\infty}\frac{z^n}{\sqrt{n!}}|n\rangle$$

characterized by a complex number z.

a) Calculate the normalization A of the coherent state $|z\rangle$. (Hint: The sum can be done to obtain a simple form for A.)

[2 points]

b) Show that the coherent state is an eigenstate of the annihilation operator and calculate the eigenvalue.

[2 points]

c) Calculate the expectation value $\mathcal{N} = \langle N\rangle$ of the number operator $N = a^\dagger a$ and the relative uncertainty $\Delta\mathcal{N}/\mathcal{N}$ in such a state.

[2 points]

d) Suppose the oscillator begins in a coherent state at time $t = 0$. Calculate the probability of finding the system in this state at a later time $t > 0$.

[2 points]

e) Following part b), determine if the evolved state is (or is not) still an eigenstate of the annihilation operator, and if so, calculate the time-dependent eigenvalue.

[2 points]

3.4 Rigid rotor with dipole moment

A quantum mechanical rigid rotor with Hamiltonian

$$\hat{H}^{(0)} = \frac{\hat{L}_z^2}{2I},$$

is constrained to rotate in the xy plane and has moment of inertia I about its axis of rotation, z. Here, $\hat{L}_z$ is the z component of the angular momentum operator, whose eigenvalues are $\hbar m$.

(a) (2 points) Find the energy eigenvalues $\varepsilon_m^{(0)}$ of $\hat{H}^{(0)}$, and the corresponding normalized wave functions $\psi_m^{(0)}(\phi)$ as a function of rotation angle ϕ. What is the range of m?

Now turn on a uniform electric field $\boldsymbol{E}$ along the x direction, and assume that the rotor has an electric dipole moment p so that a perturbation term

$$\hat{H}^{(1)} = -pE\cos\phi$$

must be added to the Hamiltonian, $\hat{H} = \hat{H}^{(0)} + \hat{H}^{(1)}$. Also assume that pE is small enough to allow expansions of the eigenfunctions ψ_m and eigenvalues ε_m of $\hat{H}$ around $pE = 0$.

(b) (2 points) Show that the first-order correction to the energy levels $\varepsilon_m^{(0)}$ of the rotor vanishes for all m.

(c) (3 points) The perturbed *ground-state* wave function ψ_0 can be approximated to first order in pE as

$$\psi_0(\phi) \approx \psi_0^{(0)}(\phi) + pE\,\psi_0^{(1)}(\phi).$$

Find $\psi_0^{(1)}$.

(d) (3 points) Calculate the energy shift of the ground-state using second-order perturbation theory.