Exam #: ______________________

Printed Name: ______________________

Signature: ______________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Unified Graduate Examination

Part III

Quantum Mechanics

Friday, September 22, 2017, 10:00 to 12:40

The examination booklet is numbered in the upper right-hand corner of the cover page. Print and then sign your name in the spaces provided on the cover page. For identification purposes, be sure to submit this page together with your answers when the exam is finished.

There are four questions, each beginning on a new page. Read all four questions before attempting any answer. You may answer as many questions as you wish, however, only your top three scores will be used in the evaluation of your performance for a Ph.D. pass in this area, or your top two scores for a master's pass.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. When you start a new problem, start a new page. Place both the exam number and the question number on all pages you wish to have graded. You are encouraged to use the constants on the following page, where appropriate, to help you solve the problems.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic and will be provided. **Personal calculators are not allowed**. Dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries are not allowed. No other papers or books may be used.**

When you have finished, come to the front of the room. For each problem, put the pages in order and paper clip them together. Then put all problems in numerical order and place them in the envelope provided. Finally, hand the envelope to the proctor.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg (0.511 MeV/c^2)
Proton rest mass (m_p)	1.673×10^{-27} kg (938 MeV/c^2)
Neutron rest mass (m_n)	1.675×10^{-27} kg (940 MeV/c^2)
Atomic mass unit (AMU)	1.66×10^{-27} kg
Atomic weight of a hydrogen atom	1 AMU
Atomic weight of a nitrogen atom	14 AMU
Atomic weight of an oxygen atom	16 AMU
Planck's constant (h)	6.63×10^{-34} J·s
Bohr Magneton (μ_B)	9.27×10^{-28} J/G
Speed of light in vacuum (c)	3.00×10^{8} m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Gravitational constant (G)	6.67×10^{-11} N·m^2/kg^2
Permeability of free space (μ_0)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space (ϵ_0)	8.85×10^{-12} F/m
Mass of earth (M_E)	5.98×10^{24} kg
Equatorial radius of earth (R_E)	6.38×10^{6} m
Mass of Sun (M_S)	1.99×10^{30} kg
Radius of Sun (R_S)	6.96×10^{8} m
Classical electron radius (r_0)	2.82×10^{-15} m
Density of water	1.0 kg/liter
Density of ice	0.917 kg/liter
Specific heat of water	4184 J/(kg K)
Specific heat of ice	2060 J/(kg K)
Latent heat of water	334.2 kJ/kg
Heat of vaporization of water	2260 kJ/kg
Specific heat of oxygen (c_V)	21.1 J/mole·K
Specific heat of oxygen (c_P)	29.4 J/mole·K
Gravitational acceleration on Earth (g)	9.8 m/s^2
1 atmosphere	1.01×10^{5} Pa

Problem 3.1

Consider a neutral spinless quantum particle moving in a (real) potential $V(\vec{r})$ and described by a wave function $\psi(\vec{r}, t)$. Conservation of probability can be expressed by a continuity equation of the form

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0.$$

(a) [5 points] Given that $\rho(\vec{r}) = |\psi(\vec{r}, t)|^2$, and given the form of the continuity equation, derive an expression for $\vec{j}$.

(b) [3 points] Using the continuity equation, show under what conditions the total probability $P = \int \rho(\vec{r}) d\vec{r}$ is conserved (i.e. constant).

(c) [2 points] If the potential is complex, how is the continuity equation modified and why is P not conserved? How could we choose a complex V such that P decreases in time?

Problem 3.2

A particle of mass m moves non-relativistically in one dimension in a potential $V(x)$ which vanishes for $|x| > L$. The spatially averaged value of $V(x)$ over the range $|x| < L$ is $-V_0$, with $V_0 > 0$ (i.e., the potential is, on average, attractive).

(a) [6 points] Show using the variational method that there must be at least one bound state of the particle in this potential. Take as your variational ansatz a wavefunction that satisfies the Schrodinger equation for this system exactly in the regions where $V(x) = 0$.

(b) [4 points] Assuming that V_0 is very small, calculate the ground state energy using the results from (a). How small does V_0 have to be for your approximations to be valid? Give a quantitative answer to this last question in terms of m, L, and $\hbar$.

Problem 3.3

Consider two identical spin-1/2 fermions each with mass m confined to move along a line of length L. The confining potential energy is zero for $0 < x < L$ and infinite elsewhere. The interaction between the two particles can be expressed by the potential energy $V(x_1, x_2)$ where x_1 and x_2 are the coordinates of the two particles. Assume that the interaction energy does not depend on the spin states of the particles.

(a) (2 points) If the interaction is zero, that is if $V(x_1, x_2) = 0$, what is the ground-state energy of the combined system? Write down the normalized ground-state wave function accounting for spatial and spin-state symmetry.

(b) (3 points) Again for $V(x_1, x_2) = 0$, what is the first excited energy level and its degeneracy? Write down the corresponding normalized wave functions, which are also eigenstates of total spin, considering spatial and spin-state symmetry.

(c) (3 points) Now consider an interaction of the form $V(x_1, x_2) = \alpha\delta(x_1 - x_2)$. If α is small, what is the correction to the ground state energy due to this interaction from first-order perturbation theory? You may leave your answer as an integral.

(d) (2 points) Considering the same perturbation as part (c), describe in words what happens to the degeneracy of the first-excited state.

3.4

A one-dimensional harmonic oscillator of mass m and natural frequency ω_0 is in its ground state. At $t = 0$, a uniform force F is applied in the x direction, and at a time $t = T$ the force is switched off.

(a) **(1 point)** Write the expression for the perturbation potential for times $0 < t < T$.

(b) **(1 point)** What is the expectation value $\langle 0|V|0\rangle$ of the perturbation potential in the ground state $|0\rangle$ of the unperturbed oscillator?

(c) **(3 points)** Calculate the matrix element $\langle 1|V|0\rangle$ of the perturbation potential between ground state and first excited state.

(d) **(2 points)** In first-order time-dependent perturbation theory, how is the matrix element $\langle 1|V|0\rangle$ related to the probability $P_{0\to 1}$ of finding the particle in the first excited state at time T?

(e) **(2 points)** Apply the relation in part **(d)** to find $P_{0\to 1}$ as a function of F and T.

(f) **(1 point)** For what value(s) of T is this probability $P_{0\to 1}$ maximal?

Hint: The ladder operators of the harmonic oscillator are related to the position operator by

$$x = \sqrt{\frac{\hbar}{2m\omega_0}}\left(a^\dagger + a\right).$$