Exam #: ________________________

Printed Name: ________________________

Signature: ________________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Unified Graduate Examination

Part II

Electromagnetism

Tuesday, April 3, 2018, 13:30 to 16:10

The examination booklet is numbered in the upper right-hand corner of the cover page. Print and then sign your name in the spaces provided on the cover page. For identification purposes, be sure to submit this page together with your answers when the exam is finished.

There are four questions, each beginning on a new page. Read all four questions before attempting any answer. You may answer as many questions as you wish, however, only your top three scores will be used in the evaluation of your performance for a Ph.D. pass in this area, or your top two scores for a master's pass.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. When you start a new problem, start a new page. Place both the exam number and the question number on all pages you wish to have graded. You are encouraged to use the constants on the following page, where appropriate, to help you solve the problems.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic and will be provided. **Personal calculators are not allowed**. Dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries are not allowed. No other papers or books may be used.**

When you have finished, come to the front of the room. For each problem, put the pages in order and staple them together. Then put all problems in numerical order and place them in the envelope provided. Finally, hand the envelope to the proctor.

Constants

Electron charge (e)	$1.60 \times 10^{-19}\,\mathrm{C}$
Electron rest mass (m_e)	$9.11 \times 10^{-31}\,\mathrm{kg}$ ($0.511\,\mathrm{MeV/c^2}$)
Proton rest mass (m_p)	$1.673 \times 10^{-27}\,\mathrm{kg}$ ($938\,\mathrm{MeV/c^2}$)
Neutron rest mass (m_n)	$1.675 \times 10^{-27}\,\mathrm{kg}$ ($940\,\mathrm{MeV/c^2}$)
Atomic mass unit (AMU)	$1.66 \times 10^{-27}\,\mathrm{kg}$
Atomic weight of a hydrogen atom	1 AMU
Atomic weight of a nitrogen atom	14 AMU
Atomic weight of an oxygen atom	16 AMU
Planck's constant (h)	$6.63 \times 10^{-34}\,\mathrm{J{\cdot}s}$
Bohr Magneton (μ_B)	$9.27 \times 10^{-28}\,\mathrm{J/G}$
Speed of light in vacuum (c)	$3.00 \times 10^{8}\,\mathrm{m/s}$
Boltzmann's constant (k_B)	$1.38 \times 10^{-23}\,\mathrm{J/K}$
Gravitational constant (G)	$6.67 \times 10^{-11}\mathrm{N{\cdot}m^2/kg^2}$
Permeability of free space (μ_0)	$4\pi \times 10^{-7}\,\mathrm{H/m}$
Permittivity of free space (ϵ_0)	$8.85 \times 10^{-12}\,\mathrm{F/m}$
Mass of earth (M_E)	$5.98 \times 10^{24}\,\mathrm{kg}$
Equatorial radius of earth (R_E)	$6.38 \times 10^{6}\,\mathrm{m}$
Mass of Sun (M_S)	$1.99 \times 10^{30}\,\mathrm{kg}$
Radius of Sun (R_S)	$6.96 \times 10^{8}\,\mathrm{m}$
Classical electron radius (r_0)	$2.82 \times 10^{-15}\,\mathrm{m}$
Density of water	1.0 kg/liter
Density of ice	0.917 kg/liter
Specific heat of water	4180 J/(kg K)
Specific heat of ice	2050 J/(kg K)
Heat of fusion of water	334 kJ/kg
Heat of vaporization of water	2260 kJ/kg
Specific heat of oxygen (c_V)	$21.1\,\mathrm{J/mole{\cdot}K}$
Specific heat of oxygen (c_P)	$29.4\,\mathrm{J/mole{\cdot}K}$
Gravitational acceleration on Earth (g)	$9.8\,\mathrm{m/s^2}$
1 atmosphere	1.01×10^{5} Pa

Problem 2.1

A nonconducting thin disk has charge uniformly distributed over its surface. The disk sits in the xy-plane and has radius R and surface charge density σ. It spins with angular velocity $\Omega_\circ$ about the z-axis.

a. (4 points) Find the magnetic field of the disk on the z-axis.

b. (3 points) Far from the disk, the magnetic field may be approximated as an ideal dipole field,

$$\mathbf{B} = \frac{\mu_\circ}{4\pi}\frac{|\mathbf{m}|}{r^3}\left(\hat{r}\ 2\cos\theta + \hat{\theta}\sin\theta\right), \tag{1}$$

whose magnetic moment is the same as that of the spinning, charged disk. Find the magnetic moment for the disk, $\mathbf{m}$.

c. (3 points) For $|\mathrm{z}|/R \gg 1$, show that the exact on-axis magnetic field and the ideal dipole field expressions agree.

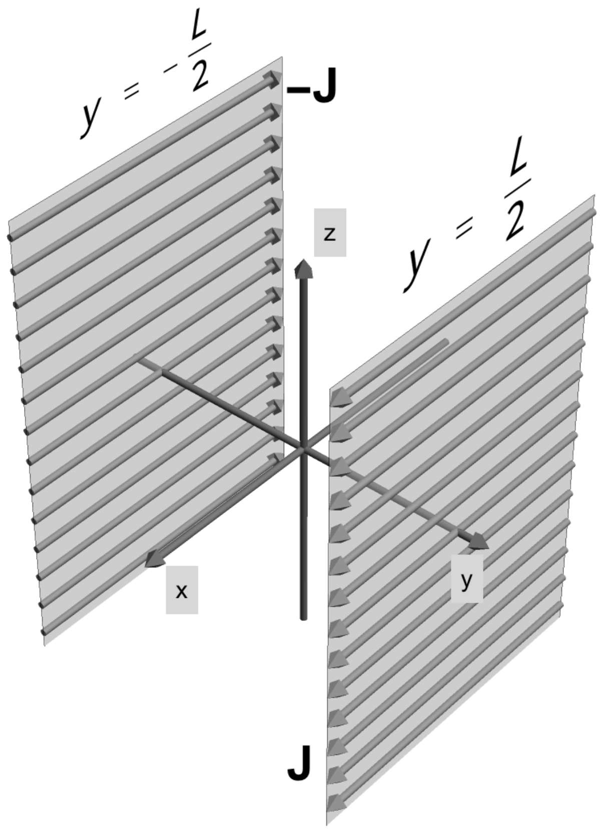
Helpful integral:

$$\int \frac{r^3 \mathrm{d}r}{(r^2 + \mathrm{z}^2)^{3/2}} = \frac{r^2 + 2\mathrm{z}^2}{\sqrt{r^2 + \mathrm{z}^2}} \tag{2}$$

2.2

Two identical thin conducting plates with two dimensional conductivity σ lie parallel to the x-z plane, at $y = \pm L/2$. Their lateral dimensions are much greater than their separation L. The "front" plate (at $y = L/2$) carries a uniform two-dimensional current density of magnitude J in the $+x$ direction; the "back" plate (at $y = -L/2$) carries a uniform two-dimensional current density of equal magnitude in the opposite direction (see figure!).

The y component of the electric field is only non-zero between the two plates, and is time independent. All of space except the two plates is vacuum.



a) Calculate the x and z components $E_{x,z}(x,z)_{\text{front,back}}$ of the $\mathbf{E}$ field on the front and back plates. (1 pt)

b) Calculate the electrostatic potential $V(x,z)_{\text{front,back}}$ on the front and back plates, assuming that $V(x=0,z)=0$ on both plates. (1 pt)

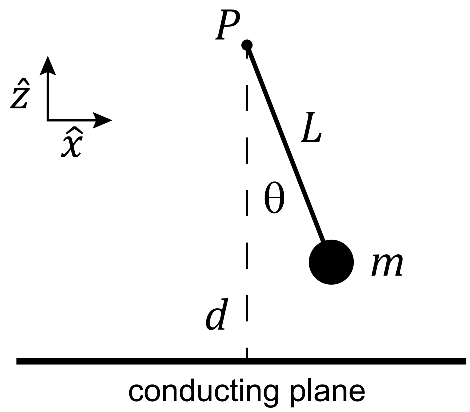
c) Calculate the electrostatic potential $V(x,y,z)$ everywhere between the two plates. (Hint: use separation of variables.) (4 pts)

d) Calculate the electric field $\mathbf{E}(x,y,z)$ everywhere between the two plates. (2 pts)

e) Calculate the surface charge density $\rho(x,z)_{\text{front,back}}$ on the front and back plates. (2 pts)

Problem 2.3

Consider a particle of mass m and charge $q > 0$ connected to a fixed point P by a rod of length L. Below the charged mass is an infinite conducting plane at a distance d from P. Assume there is no gravity and denote the angle the rod makes with vertical as θ. Let the coordinate system be defined such that P is along the $\hat{z}$ axis and that the charge moves along the $\hat{x}$ direction.



(a) (1 points) Where is the equilibrium position of the mass?

(b) (3 points) What is the potential field of this system in Cartesian coordinates, ignoring the retarded components of the potential. In terms of L and θ, what are the conditions that allow us to ignore those components?

(c) (4 points) Find the frequency of small oscillations about the equilibrium.

(d) (2 points) Neglecting any mechanical effects, like friction or air resistance, do you expect this charged mass to oscillate indefinitely? Explain your answer concisely in words.

2.4

Consider a sphere of radius a made of a uniform magnetizable material with magnetic permeability μ, surrounded by air of permeability μ_0.

A uniform external magnetic field B_0 parallel to the z axis is added.

(a) **(2 points)** Because there are no currents inside or outside the sphere (excluding the surface), the magnetic field in each region must solve Maxwell's equations for a uniform medium with the appropriate permeability (μ or μ_0). In addition to a uniform field, these equations are also solved by the field of a magnetic point dipole which has the following form in spherical coordinates:

$$\boldsymbol{B}_{\text{dipole}} = \frac{C}{r^3}\left(2\cos\theta\,\hat{\boldsymbol{r}} + \sin\theta\,\hat{\boldsymbol{\theta}}\right). \tag{1}$$

Here, $\hat{\boldsymbol{r}}$ and $\hat{\boldsymbol{\theta}}$ are the radial and polar unit vectors, respectively, and $r > 0$ is assumed. C is a constant to be determined below.
To make use of the spherical geometry, write the spherical-coordinate expression for a uniform magnetic field, $B_0\,\hat{z}$, using the same basis vectors as in eq. (1).

(b) **(3 points)** Leaving the constant C undetermined for now, what is the most general form of the magnetic field $\boldsymbol{B}$ in spherical coordinates for the two cases, $r < a$ and $r > a$?

(c) **(2 points)** From Maxwell's equations $\nabla \cdot \boldsymbol{B} = 0$ and $\nabla \times \boldsymbol{H} = 0$ (valid because there are no free currents), we can deduce relations between the fields inside and outside the surface as $r \to a$. Find one such equation for a spherical component of $\boldsymbol{B}$, such that no additional unknowns are introduced, beyond those already present in part **(b)**. Do the same for one of the spherical components of $\boldsymbol{H}$.

(d) **(3 points)** Find the value of C in (1) in terms of μ and the external field B_0. This measures the induced magnetic dipole moment of the sphere.