Exam #: ______________________

Printed Name: ______________________

Signature: ______________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Unified Graduate Examination

Part II

Electromagnetism

Tuesday, April 4, 2017, 13:30 to 16:10

The examination booklet is numbered in the upper right-hand corner of the cover page. Print and then sign your name in the spaces provided on the cover page. For identification purposes, be sure to submit this page together with your answers when the exam is finished.

There are four questions, each beginning on a new page. Read all four questions before attempting any answer. You may answer as many questions as you wish, however, only your top three scores will be used in the evaluation of your performance for a Ph.D. pass in this area, or your top two scores for a master's pass.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. When you start a new problem, start a new page. Place both the exam number and the question number on all pages you wish to have graded. You are encouraged to use the constants on the following page, where appropriate, to help you solve the problems.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic and will be provided. **Personal calculators are not allowed**. Dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries are not allowed. No other papers or books may be used.**

When you have finished, come to the front of the room. For each problem, put the pages in order and staple them together. Then put all problems in numerical order and place them in the envelope provided. Finally, hand the envelope to the proctor.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg (0.511 MeV/c^2)
Proton rest mass (m_p)	1.673×10^{-27} kg (938 MeV/c^2)
Neutron rest mass (m_n)	1.675×10^{-27} kg (940 MeV/c^2)
Atomic mass unit (AMU)	1.66×10^{-27} kg
Atomic weight of a hydrogen atom	1 AMU
Atomic weight of a nitrogen atom	14 AMU
Atomic weight of an oxygen atom	16 AMU
Planck's constant (h)	6.63×10^{-34} J·s
Bohr Magneton (μ_B)	9.27×10^{-28} J/G
Speed of light in vacuum (c)	3.00×10^{8} m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Gravitational constant (G)	6.67×10^{-11}N·m^2/kg^2
Permeability of free space (μ_0)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space (ϵ_0)	8.85×10^{-12} F/m
Mass of earth (M_E)	5.98×10^{24} kg
Equatorial radius of earth (R_E)	6.38×10^{6} m
Mass of Sun (M_S)	1.99×10^{30} kg
Radius of Sun (R_S)	6.96×10^{8} m
Classical electron radius (r_0)	2.82×10^{-15} m
Density of water	1.0 kg/liter
Density of ice	0.917 kg/liter
Specific heat of water	4180 J/(kg K)
Specific heat of ice	2050 J/(kg K)
Heat of fusion of water	334 kJ/kg
Heat of vaporization of water	2260 kJ/kg
Specific heat of oxygen (c_V)	21.1 J/mole·K
Specific heat of oxygen (c_P)	29.4 J/mole·K
Gravitational acceleration on Earth (g)	9.8 m/s^2
1 atmosphere	1.01×10^{5} Pa

Pauli spin matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad , \; \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad , \; \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad .$$

Stirling's formula

$$\log(N!) \approx N \log N - N$$

Integrals

$$\int_{-\infty}^{\infty} e^{-ax^2} \mathrm{d}x = \left(\frac{\pi}{a}\right)^{1/2}$$

$$\int_{-\infty}^{\infty} x^2 \; e^{-ax^2} \mathrm{d}x = \frac{1}{2a} \left(\frac{\pi}{a}\right)^{1/2}$$

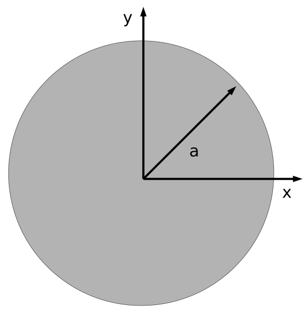
Differential Operators

$$\nabla^2_{\text{spherical}} = \left[\frac{1}{r^2}\partial_r(r^2\partial_r) + \frac{1}{r^2 \sin\theta}\partial_\theta(\sin\theta\partial_\theta) + \frac{1}{r^2\sin^2\theta}\partial^2_\phi\right]$$

$$\nabla^2_{\text{cylindrical}} = \left[\frac{1}{r}\partial_r(r\partial_r) + \frac{1}{r^2}\partial^2_\phi + \partial^2_z\right]$$

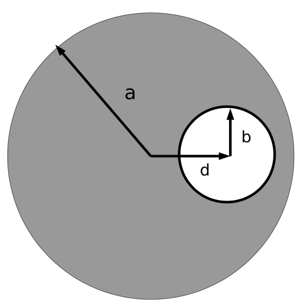
2.1

Consider an infinitely long, solid, cylindrical wire with radius a that has uniform current density $\boldsymbol{j}$, and carries a total current I. The wire is coaxial with the z-axis and the current flows in the $+z$ direction.



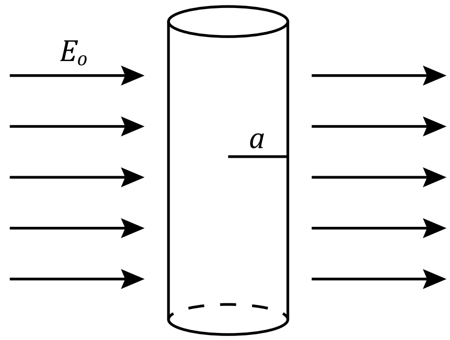
(a) Find the magnetic field $\boldsymbol{B}$ inside and outside the wire. Give your answer in cylindrical coordinates.

(b) Suppose the wire has a (non-conducting) hole of radius b offset from the axis of the wire by distance d as shown in the figure ($b + d < a$). If the current density is uniform outside the hole, and the total current I has the same value as in part **(a)**, find the magnetic field $\boldsymbol{B}$ inside the hole. Give your result in Cartesian coordinates.



Problem 2.2

Consider an infinitely long, grounded conducting cylinder of radius a in a uniform external electric field $\mathbf{E}_o$, perpendicular to the axis of the cylinder.



a) (1 point) What is the potential on the surface of the cylinder?

b) (5 points) Find an expression for the potential outside the cylinder, and verify that this reduces to the expected potential far from the cylinder.

c) (2 points) What is the electric field outside the cylinder?

d) (2 points) Find an expression for the surface charge induced on the cylinder.

Problem 2.3

Consider a set of four charges $Q = \pm q$, arranged in a square, stuck to a plane, with the diagonal distance between like-sign charges to be a as shown in the Figure.

a) Suppose a gravitational wave passes through the set of charges perpendicular to the plane. Assume the effect of the gravitational wave at one moment in time t_0 is to lengthen (shorten) the distances between the positive (negative) charges by an amount $+\epsilon$ $(-\epsilon)$. Calculate the change in the electrostatic potential energy $\Delta U = U(t_0) - U(t = -\infty)$.

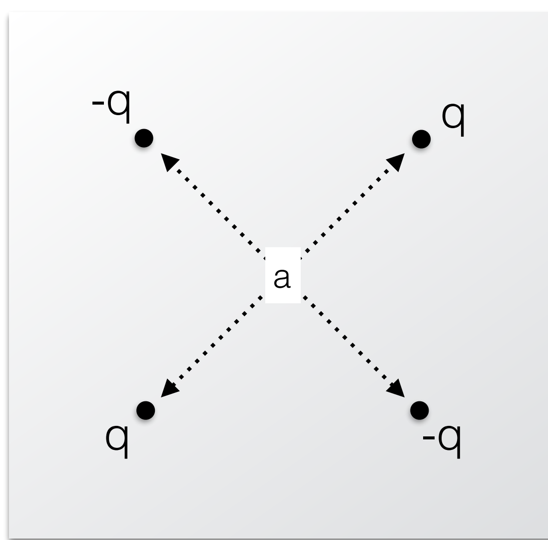
[3 points]

b) Calculate the moments q_{lm} of a multipole expansion up to $l = 2$. If a particular moment evaluates to zero, explain why.

[4 points]

c) Calculate the leading contribution to the electric potential at a distance $r \gg a$ far from the set of charges. (You can leave your result in terms of the appropriate spherical harmonic functions.) Repeat part a) to determine the change to the electric potential $\Delta\Phi(\mathbf{x}) = \Phi(\mathbf{x})|_{t_0} - \Phi(\mathbf{x})_{t=-\infty}$.

[3 points]



Useful formuale:

Electric potential in multipole expansion:

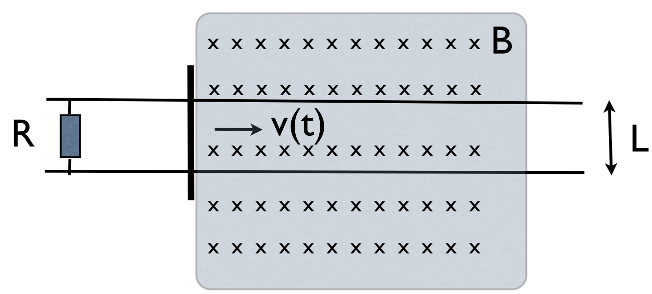
$$\Phi(\mathbf{x}) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \frac{4\pi}{2l+1} q_{lm} \frac{Y_{lm}(\theta, \phi)}{r^{l+1}}$$

The first few multipole moments in cartesian coordinates in terms of the total charge q_{tot}, the dipole moment $\mathbf{p} \equiv \int \mathbf{x}' \rho(\mathbf{x}') d^3x'$ and the traceless quadrupole moment tensor $Q_{ij} \equiv \int (3x_i'x_j' - r'^2\delta_{ij})\rho(\mathbf{x}')d^3x'$:

$$\begin{aligned}
q_{00} &= \frac{1}{\sqrt{4\pi}} q_{\text{tot}} \\
q_{11} &= -\sqrt{\frac{3}{8\pi}} (p_x - ip_y) \\
q_{10} &= \sqrt{\frac{3}{4\pi}} p_z \\
q_{22} &= \frac{1}{12}\sqrt{\frac{15}{2\pi}} (Q_{11} - 2iQ_{12} - Q_{22}) \\
q_{21} &= -\frac{1}{3}\sqrt{\frac{15}{8\pi}} (Q_{13} - iQ_{23}) \\
q_{20} &= \frac{1}{2}\sqrt{\frac{5}{4\pi}} Q_{33} \\
&\text{and} \\
q_{l,-m} &= (-1)^m q_{lm}^*
\end{aligned}$$

Problem 2.4

A conducting rod of mass m slides on frictionless conducting rails into a region of constant magnetic field with magnitude B that points into the page (see Figure). The rails are connected to a resistor R on the left and are a distance L apart. Rod and rails have negligible resistance. Initially the rod has velocity $v(0)$.



(a) Find the velocity $v(t)$ of the rod (as long as it is in the magnetic field) in two different ways:

(i) [4 points] by making use of energy conservation (how much energy is dissipated in the resistor as a function of $v(t)$?)

(ii) [4 points] by making use of the Lorentz force on the rod.

(b) [2 points] Explain in what direction the current flows through the resistor and rod.