Exam #: ________________________

Printed Name: ________________________

Signature: ________________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Unified Graduate Examination

Part II

Electromagnetism

Monday, March 28, 2016, 13:30 to 16:10

The examination booklet is numbered in the upper right-hand corner of the cover page. Print and then sign your name in the spaces provided on the cover page. For identification purposes, be sure to submit this page together with your answers when the exam is finished.

There are four questions, each beginning on a new page. Read all four questions before attempting any answer. You may answer as many questions as you wish, however, only your top three scores will be used in the evaluation of your performance for a Ph.D. pass in this area, or your top two scores for a master's pass. (These scores will be added to your aggregate if taking the exam "under the old rules".)

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. When you start a new problem, start a new page. Place both the exam number and the question number on all pages you wish to have graded. You are encouraged to use the constants on the following page, where appropriate, to help you solve the problems.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic and will be provided. **Personal calculators are not allowed**. Dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries are not allowed. No other papers or books may be used.**

When you have finished, come to the front of the room. For each problem, put the pages in order and staple them together. Then put all problems in numerical order and place them in the envelope provided. Finally, hand the envelope to the proctor.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg (0.511 MeV/c^2)
Proton rest mass (m_p)	1.673×10^{-27} kg (938 MeV/c^2)
Neutron rest mass (m_n)	1.675×10^{-27} kg (940 MeV/c^2)
Atomic mass unit (AMU)	1.66×10^{-27} kg
Atomic weight of a hydrogen atom	1 AMU
Atomic weight of a nitrogen atom	14 AMU
Atomic weight of an oxygen atom	16 AMU
Planck's constant (h)	6.63×10^{-34} J·s
Bohr Magneton (μ_B)	9.27×10^{-28} J/G
Speed of light in vacuum (c)	3.00×10^{8} m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Gravitational constant (G)	6.67×10^{-11} N·m^2/kg^2
Permeability of free space (μ_0)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space (ϵ_0)	8.85×10^{-12} F/m
Mass of earth (M_E)	5.98×10^{24} kg
Equatorial radius of earth (R_E)	6.38×10^{6} m
Mass of Sun (M_S)	1.99×10^{30} kg
Radius of Sun (R_S)	6.96×10^{8} m
Classical electron radius (r_0)	2.82×10^{-15} m
Density of water	1.0 kg/liter
Density of ice	0.917 kg/liter
Specific heat of water	4180 J/(kg K)
Specific heat of ice	2050 J/(kg K)
Heat of fusion of water	334 kJ/kg
Heat of vaporization of water	2260 kJ/kg
Specific heat of oxygen (c_V)	21.1 J/mole·K
Specific heat of oxygen (c_P)	29.4 J/mole·K
Gravitational acceleration on Earth (g)	9.8 m/s^2
1 atmosphere	1.01×10^{5} Pa

Problem 1

Two rings each of mass m and radius b have magnetic dipole moments of magnitude μ. The rings are placed on a nonconducting, frictionless pole aligned with the z-axis. Gravity is uniform and given by $\vec{g} = -g\vec{e}_z$. The dipole moment for the bottom ring points in the +z direction and the dipole moment for the top ring points in the -z direction. The bottom ring is fixed to the pole while the top ring is free to slide up and down the pole.

(a) (3 points) Plot the magnetic interaction energy of the upper dipole with the magnetic field of the bottom dipole as a function of the height z.

(b) (4 points) Find the equilibrium height h at which the top ring floats above the bottom ring. Assume that $b \ll h$.

(c) (3 points) Find the frequency of small oscillations for the top ring if it oscillates about its equilibrium height, h. Ignore effects of induction and radiation.

Problem 2

A capacitor C has been charged up to potential V_0 ; at time $t = 0$ it is connected to a resistor R, and begins to discharge.

(a) **(2 points)** Determine the charge of the capacitor as a function of time, $Q(t)$. What is the current through the resistor, $I(t)$?

(b) **(3 points)** What was the original energy stored in the capacitor? By integrating the Joule heating law, $P = VI = I^2R$, confirm that the heat delivered to the resistor is equal to the energy lost by the capacitor.

Now imagine charging up an identical capacitor by connecting it in series with the same resistor to a battery of fixed voltage V_0 at time $t = 0$.

(c) **(3 points)** Again determine $Q(t)$ and $I(t)$.

(d) **(2 points)** Find the total energy output of the battery ($\int V_0 I dt$). Determine the heat delivered to the resistor.

Problem 3

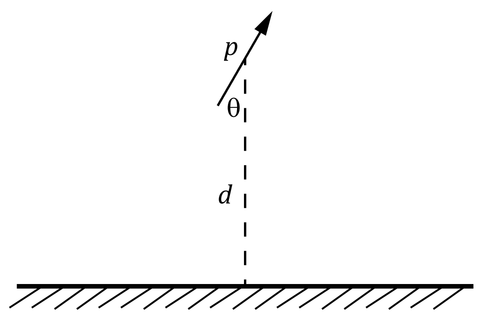
A point charge q sits a distance d along the $+\hat{z}$ direction from a grounded infinite plate, immediately above the coordinate origin.

a) (4 points) What is the potential ϕ and what is the force between the point charge and the plate?

b) (3 points) What is the surface charge density on the plate? Verify that the surface charge density integrates to $-q$.

c) (3 points) If the point charge is replaced by a point dipole of magnitude p that is allowed to freely rotate in the plane shown below, how many equilibrium points are there as a function of θ? Which of those are stable? Recall that the electrostatic dipole-dipole interaction energy is

$$U = \frac{\bar{p} \cdot \bar{p}' - 3(\bar{p} \cdot \hat{r})(\bar{p}' \cdot \hat{r})}{r^3}$$



Problem 4

A parallel-plate capacitor with plate area A and plate separation $d \ll \sqrt{A}$ is charged with a battery to a potential V_0. The battery is disconnected and a dielectric slab of thickness d and dielectric constant $\kappa > 1$ is slid between the plates such that the volume between the capacitor plates is completely filled with the dielectric. Assuming friction is negligible, calculate the total work done on the dielectric slab.