Exam #: ______________________

Printed Name: ______________________

Signature: ______________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Unified Graduate Examination

Part II

Electromagnetism

Monday, September 28, 2015, 13:30 to 16:10

The examination booklet is numbered in the upper right-hand corner of the cover page. Print and then sign your name in the spaces provided on the cover page. For identification purposes, be sure to submit this page together with your answers when the exam is finished.

There are four questions, each beginning on a new page. Read all four questions before attempting any answer. You may answer as many questions as you wish, however, only your top three scores will be used in the evaluation of your performance for a Ph.D. pass in this area, or your top two scores for a master's pass. (These scores will be added to your aggregate if taking the exam "under the old rules".)

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. When you start a new problem, start a new page. Place both the exam number and the question number on all pages you wish to have graded. You are encouraged to use the constants on the following page, where appropriate, to help you solve the problems.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic and will be provided. **Personal calculators are not allowed**. Dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries are not allowed. No other papers or books may be used.**

When you have finished, come to the front of the room. For each problem, put the pages in order and staple them together. Then put all problems in numerical order and place them in the envelope provided. Finally, hand the envelope to the proctor.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg (0.511 MeV/c^2)
Proton rest mass (m_p)	1.673×10^{-27} kg (938 MeV/c^2)
Neutron rest mass (m_n)	1.675×10^{-27} kg (940 MeV/c^2)
Atomic mass unit (AMU)	1.66×10^{-27} kg
Atomic weight of a hydrogen atom	1 AMU
Atomic weight of a nitrogen atom	14 AMU
Atomic weight of an oxygen atom	16 AMU
Planck's constant (h)	6.63×10^{-34} J·s
Bohr Magneton (μ_B)	9.27×10^{-24} J/T
Speed of light in vacuum (c)	3.00×10^{8} m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Gravitational constant (G)	6.67×10^{-11} N·m^2/kg^2
Permeability of free space (μ_0)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space (ϵ_0)	8.85×10^{-12} F/m
Classical electron radius (r_0)	2.82×10^{-15} m
Density of water	1.0 kg/liter
Density of ice	0.917 kg/liter
Specific heat of water	4180 J/(kg K)
Specific heat of ice	2050 J/(kg K)
Heat of fusion of water	334 kJ/kg
Heat of vaporization of water	2260 kJ/kg
Specific heat of oxygen (c_V)	21.1 J/mole·K
Specific heat of oxygen (c_P)	29.4 J/mole·K
Gravitational acceleration on Earth (g)	9.8 m/s^2
1 atmosphere	1.01×10^{5} Pa

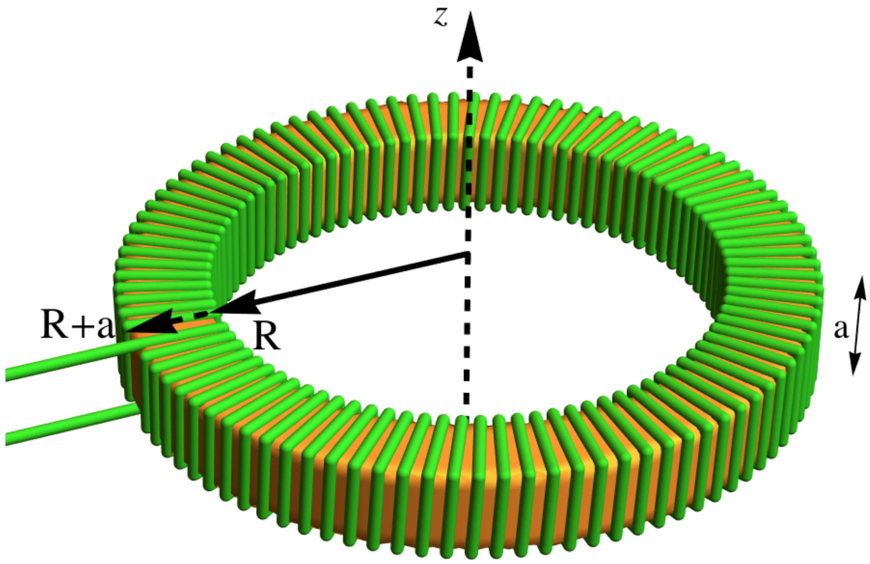
Problem 1

a. (2 points) Calculate the electric force that acts on one plate of a parallel-plate capacitor given the potential difference between the plates, $\Delta\phi$. Assume that the plates are square with side length L and separated by distance d. Ignore edge effects in this problem.

b. (4 points) If the plates are insulated, how much external work could be done by letting the plates come together?

c. (4 points) Calculate the energy stored in the electric field of the capacitor described in Part a. How does this energy compare to the work found in Part b?

Problem 2



An iron ring is wrapped with N turns of wire carrying current I. The ring has the shape obtained by rotating a square of side a about the z axis, located at a distance R from the nearest side of the square. That is, the ring occupies the volume specified in cartesian coordinates by $0 < z < a$, $R < \sqrt{x^2 + y^2} < R + a$. Assume that iron exhibits an approximately linear relation between magnetization and applied field, as specified by relative permeability μ_r. Also assume that the wire is wrapped so tightly around the core that the current density is uniform and has no azimuthal component (the figure shows loosely wound turns only for clarity).

a. (3 points) Show that the field in the ring must be circumferential, concentric circles, and axially symmetric. For this part, you need not find the magnitude of the field.

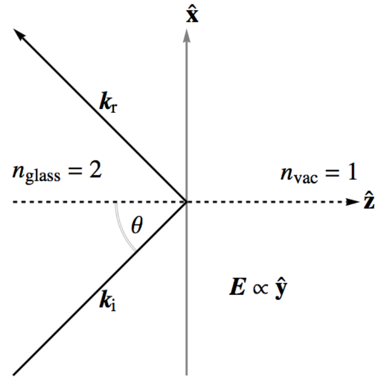
b. (2 points) Calculate the magnetic field H and the magnetic induction B in the ring as a function of the distance r from the axis.

c. 4 points) Calculate the inductance L of the ring.

d. (1 point) In the limit of a very thin ring, what is the inductance in terms of R and a, to lowest non-vanishing order?

Problem 3

A wave of frequency ω travels in a special glass of refractive index $n_{glass} = 2$, until it meets a planar boundary at $z = 0$, with vacuum for $z > 0$.



The incident wave: (i) propagates through the glass in the xz plane; (ii) is linearly polarized in the y direction; and (iii) travels at angle θ with respect to the normal direction $\hat{\mathbf{z}}$, to the glass-vacuum interface,

$$\boldsymbol{E}_\mathrm{i}(\mathbf{r}, t) = E_\mathrm{i}\, \hat{\mathbf{y}}\, e^{i(\boldsymbol{k}_\mathrm{i}\cdot\mathbf{r} - \omega t)}. \tag{1}$$

The incident angle θ is greater than the critical angle, so total internal reflection occurs. Denote the reflected wave inside the glass by

$$\boldsymbol{E}_\mathrm{r}(\mathbf{r}, t) = E_\mathrm{r}\, \hat{\mathbf{y}}\, e^{i(\boldsymbol{k}_\mathrm{r}\cdot\mathbf{r} - \omega t)}. \tag{2}$$

The subscripts i and r denote incident and reflected waves. The total field inside the glass is then $\boldsymbol{E}_\mathrm{glass}(\mathbf{r}, t) = \boldsymbol{E}_\mathrm{i}(\mathbf{r}, t) + \boldsymbol{E}_\mathrm{r}(\mathbf{r}, t)$. The electric field for $z > 0$ may be written with real numbers k_t and κ as

$$\boldsymbol{E}_\mathrm{vac}(\mathbf{r}, t) = E_\mathrm{vac}\, \hat{\mathbf{y}}\, e^{i(k_\mathrm{t} x - \omega t)} e^{-\kappa z}. \tag{3}$$

a. (1 point) Without doing any calculations, determine the y component of the *magnetic* field in each region. Justify your answer in one or two sentences.

b. (1 point) In terms of the parameters given in $\boldsymbol{E}_\mathrm{vac}(\mathbf{r}, t)$, determine the magnetic field $\boldsymbol{B}_\mathrm{vac}(\mathbf{r}, t)$ in the vacuum region.

c. (2 points) Use Stokes' theorem to show that the tangential component of the electric field must be continuous at the dielectric interface. Hint: use a a rectangular loop in the xz plane, bisected by the interface.

d. (2 points) Deduce k_t by imposing the continuity condition from Part c. Write the result in terms of θ, n_glass, ω and the speed of light, c.

e. (2 points) Using the result of Part a, write κ in terms of θ, n_glass, ω, c.

f. (2 points) Using the interface conditions for the tangential components of the electric and magnetic field, derive (in terms of θ and n_{glass}) the phase shift δ between the incident wave and the field in the vacuum, defined as

$$\frac{E_{\text{vac}}}{E_{\text{i}}} = \left|\frac{E_{\text{vac}}}{E_{\text{i}}}\right| e^{i\delta} \text{ for } z = 0.$$

Problem 4

A charge q with mass m is released from rest a distance D from an infinite, grounded, conducting plane. Ignore gravity and radiation.

a. (3 points) Write the non-relativistic equation of motion for the particle denoting distance from the plane by x.

b. (2 points) Derive an expression for the initial energy of the system.

c. (3 points) Determine how long it takes the charge to reach distance $D/2$ from the plane.

d. (2 points) Find an inequality involving the quantities q, m, and D that guarantees the validity of the non-relativistic equation of motion.

Useful integral:

$$\int \sqrt{\frac{x}{a-x}}\mathrm{d}x = -\sqrt{x(a-x)} - a\,\tan^{-1}\frac{\sqrt{x(a-x)}}{x-a} \tag{4}$$