Exam #: ________________________

Printed Name: ________________________

Signature: ________________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Unified Graduate Examination

PART II

Tuesday, September 30, 2014, 13:00 to 17:00

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

You are encouraged to use the constants and other information on the following page, where appropriate, to help you solve the problems.

There are six equally weighted questions, each beginning on a new page. Read all six questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. When you start a new problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic and will be provided. **Personal calculators are not allowed**. Dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries are not allowed. No other papers or books may be used.**

When you have finished, come to the front of the room. For each problem, put the pages in order and staple them together. Then put all problems in numerical order and place them in the envelope provided. Finally, hand the envelope to the proctor.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand in your exam paper on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	$1.60 \times 10^{-19}\,\mathrm{C}$
Electron rest mass (m_e)	$9.11 \times 10^{-31}\,\mathrm{kg}$ ($0.511\,\mathrm{MeV/c^2}$)
Proton rest mass (m_p)	$1.673 \times 10^{-27}\,\mathrm{kg}$ ($938\,\mathrm{MeV/c^2}$)
Neutron rest mass (m_n)	$1.675 \times 10^{-27}\,\mathrm{kg}$ ($940\,\mathrm{MeV/c^2}$)
Atomic mass unit (AMU)	$1.7 \times 10^{-27}\,\mathrm{kg}$
Atomic weight of a hydrogen atom	1 AMU
Atomic weight of a nitrogen atom	14 AMU
Atomic weight of an oxygen atom	16 AMU
Planck's constant (h)	$6.63 \times 10^{-34}\,\mathrm{J{\cdot}s}$
Speed of light in vacuum (c)	$3.00 \times 10^{8}\,\mathrm{m/s}$
Boltzmann's constant (k_B)	$1.38 \times 10^{-23}\,\mathrm{J/K}$
Gravitational constant (G)	$6.67 \times 10^{-11}\mathrm{N{\cdot}m^2/kg^2}$
Permeability of free space (μ_0)	$4\pi \times 10^{-7}\,\mathrm{H/m}$
Permittivity of free space (ϵ_0)	$8.85 \times 10^{-12}\,\mathrm{F/m}$
Mass of earth (M_E)	$5.98 \times 10^{24}\,\mathrm{kg}$
Equatorial radius of earth (R_E)	$6.38 \times 10^{6}\,\mathrm{m}$
Mass of Sun (M_S)	$1.99 \times 10^{30}\,\mathrm{kg}$
Radius of Sun (R_S)	$6.96 \times 10^{8}\,\mathrm{m}$
Classical electron radius (r_0)	$2.82 \times 10^{-15}\,\mathrm{m}$
Density of water	1.0 kg/liter
Density of ice	0.917 kg/liter
Specific heat of water	4180 J/(kg K)
Specific heat of ice	2050 J/(kg K)
Heat of fusion of water	334 kJ/kg
Heat of vaporization of water	2260 kJ/kg
Specific heat of oxygen (c_V)	21.1 J/mole·K
Specific heat of oxygen (c_P)	29.4 J/mole·K
Gravitational acceleration on Earth (g)	$9.8\,\mathrm{m/s^2}$
1 atmosphere	1.01×10^{5} Pa

Pauli spin matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \;,\; \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \;,\; \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \;.$$

Stirling's formula

$$\log(N!) \approx N \log N - N$$

Integrals

$$\int_{-\infty}^{\infty} e^{-ax^2} \mathrm{d}x = \left(\frac{\pi}{a}\right)^{1/2}$$

$$\int_{-\infty}^{\infty} x^2\, e^{-ax^2} \mathrm{d}x = \frac{1}{2a} \left(\frac{\pi}{a}\right)^{1/2}$$

Problem 1

Suppose an electron is (in spherical coordinates) described by the wave function

$$\Psi(r,\theta,\phi) = R(r)\left[\sqrt{\frac{3}{4}}Y_2^2(\theta,\phi)\chi_- + \frac{1}{2}Y_2^{-1}(\theta,\phi)\chi_+\right],$$

where $R(r)$ is a normalized radial function, $Y_2^2(\theta,\phi)$ and $Y_2^{-1}(\theta,\phi)$ are spherical harmonics, and χ_+ and χ_- are spinors for spin up and spin down, respectively.

a. Write down an expression for the probability of finding this particle in a spin up state at a distance from the origin lying between r and $r+dr$.

b. Write down an expression for the probability of finding this particle at a distance from the origin lying between r and $r+dr$ *irrespective* of its spin state.

c. If you measure the z component of total angular momentum, $L_z + S_z$, for this state, what values might you get and what is the probability for each?

Problem 2

An infinitely deep potential well,

$$V(x) = \begin{cases} \infty \text{ for } x < 0, \\ 0 \ \text{ for } 0 \leq x \leq a, \\ \infty \text{ for } x > a \end{cases} \tag{1}$$

contains a particle of mass m.

a. Starting from the time-independent Schrödinger equation, determine the normalized wave function of the general energy eigenstate.

b. What are the energy levels of this system?

c. Suppose the particle was in the ground state when the width of the potential well *suddenly* expanded from a to $2a$. What is the probability that the particle will be found in the (new) ground state when its energy is measured?

Problem 3

Consider a two-state atom, with energy levels $|1\rangle$ (with energy 0) and $|2\rangle$ (with energy $\hbar\omega_0$). Suppose this atom interacts with a near-resonant classical electromagnetic field $E(t) = E_0 \cos(\omega t)$. The Hamiltonian describing this interaction is $\hat{H} = \hat{\mu} E(t)$, where the nonzero matrix elements of the operator $\hat{\mu}$ are $\langle 1|\hat{\mu}|2\rangle = \langle 2|\hat{\mu}|1\rangle = \mu$.

a. Write down the Hamiltonian of the system in a 2×2 matrix representation.

b. Assume the field is turned on at $t = 0$, at which time the atom is in $|1\rangle$. Use time-dependent perturbation theory to find the probability for the atom to be found in the upper state after a short time interval τ. (You will find that the contribution from nonresonant terms is much smaller than that from resonant terms.)

c. For near-resonant excitation, perturbation theory becomes inadequate for sufficiently long times. Ignore the nonresonant terms and determine—without using perturbation theory— the upper state population as a function of time when the field is exactly resonant with the atomic transition.

Problem 4

Consider a gas of N noninteracting spin-1/2 fermions at temperature $T = 0$ uniformly spread over an area A.

a. Find the Fermi energy, ϵ_F.

b. Show that the total energy is given by

$$E = \frac{1}{2} N \epsilon_F$$

Problem 5

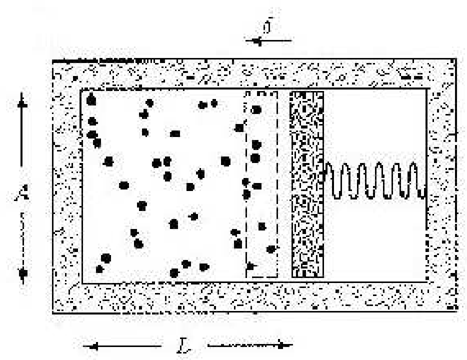
Write an expression for the partition function for N distinct classical particles, each of mass m, moving independently in identical potentials $\nu(\vec{r}_n)$ where $\vec{r}_n$ is the position of the n-th particle. Calculate the partition function in the case where the potentials are harmonic, $\nu(\vec{r}) = 0.5\mathrm{k}r^2$, where k is a constant. In this model how does the average volume occupied by each particle vary with temperature?

Problem 6

Except for an additive constant that does not depend upon energy or volume, the entropy of an ideal gas is

$$S(E, V) = \mathrm{k_B} N \ln(E^{3/2}) + \mathrm{k_B} N \ln(V)$$

where E is the internal energy, V is the volume, N is the number of particles, and $\mathrm{k_B}$ is the Boltzmann constant. The figure below shows a thermally isolated system consisting of an ideal gas with number of particles N, volume V, internal energy E, and a spring-loaded piston with cross sectional area A. At first, the piston is clamped at position L. The piston



is then unclamped and allowed to move an infinitesimally small distance δ to the left, where the piston is clamped again. During the motion, the spring pushes with a force f on the piston.

a. Give an expression for the change of potential energy stored by the spring. Where does the energy go?

b. Using the expression for entropy above, derive an expression for the change in entropy of the system, ΔS.

c. Use the result of part b to find the piston's equilibrium position L_{eq}, *i.e.*, the position at which the piston would come to rest if unclamped.

d. Show that your result for part c is in agreement with the ideal gas law.