Exam #: ________________________

Printed Name: ________________________

Signature: ________________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Unified Graduate Examination

PART II

Tuesday, October 1, 2013, 13:00 to 17:00

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

You are encouraged to use the constants and other information on the following two pages where appropriate to help you solve the problems.

There are six equally weighted questions, each beginning on a new page. Read all six questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. When you start a new problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic and will be provided. **Personal calculators are not allowed**. Dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries are not allowed. No other papers or books may be used.**

When you have finished, come to the front of the room. For each problem, put the pages in order and staple them together. Then put all problems in numerical order and place them in the envelope provided. Finally, hand the envelope to the proctor.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand in your exam paper on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg ($0.511\,\mathrm{MeV/c^2}$)
Proton rest mass (m_p)	1.673×10^{-27} kg ($938\,\mathrm{MeV/c^2}$)
Neutron rest mass (m_n)	1.675×10^{-27} kg ($940\,\mathrm{MeV/c^2}$)
Atomic mass unit (AMU)	1.7×10^{-27} kg
Atomic weight of a hydrogen atom	1 AMU
Atomic weight of a nitrogen atom	14 AMU
Atomic weight of an oxygen atom	16 AMU
Planck's constant (h)	6.63×10^{-34} J·s
Speed of light in vacuum (c)	3.00×10^{8} m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Gravitational constant (G)	$6.67 \times 10^{-11}\,\mathrm{N{\cdot}m^2/kg^2}$
Permeability of free space (μ_0)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space (ϵ_0)	8.85×10^{-12} F/m
Mass of earth (M_E)	5.98×10^{24} kg
Equatorial radius of earth (R_E)	6.38×10^{6} m
Mass of Sun (M_S)	1.99×10^{30} kg
Radius of Sun (R_S)	6.96×10^{8} m
Classical electron radius (r_0)	2.82×10^{-15} m
Density of water	1.0 kg/liter
Density of ice	0.917 kg/liter
Specific heat of water	4180 J/(kg K)
Specific heat of ice	2050 J/(kg K)
Heat of fusion of water	334 kJ/kg
Heat of vaporization of water	2260 kJ/kg
Specific heat of oxygen (c_V)	21.1 J/mole·K
Specific heat of oxygen (c_P)	29.4 J/mole·K
Gravitational acceleration on Earth (g)	$9.8\,\mathrm{m/s^2}$
1 atmosphere	1.01×10^{5} Pa

Pauli spin matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \;,\; \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \;,\; \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \;.$$

Stirling's formula

$$\log(N!) \approx N \log N - N$$

Expansion

$$\frac{1}{|r+x|} \approx \frac{1}{r} - \frac{\vec{r}\cdot\vec{x}}{r^3} - \frac{|\vec{x}|^2}{2r^3} + \frac{3(\vec{x}\cdot\vec{r})^2}{2r^5} \quad \text{for} \;\; |\vec{x}| \ll r$$

Problem 1

Consider a set of three spin $\frac{1}{2}$ particles prepared in a state $|\Psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\uparrow\rangle - |\downarrow\downarrow\downarrow\rangle)$, where $|\uparrow\rangle$ represents a spin in the +z direction. The three spins are sent to three different scientists (first spin to first scientist, second spin to second scientist, third spin to third scientist), who have the option to measure either spin observable $\mathrm{X} = \sigma_x$ or $\mathrm{Y} = \sigma_y$. For the first few steps, assume that standard quantum mechanical measurements are correct.

(a) For a given scientist measuring X or Y on this wave function what are the possible measurement outcomes?

(b) Now consider that the three scientists are all measuring either X or Y for each state. They compare their results for each state by multiplying their measurements. There are four types of experiments (due to cyclic symmetry) where 1) all scientists measure X, 2) all scientists measure Y, 3) two scientists measure X, one measures Y, and 4) 2 scientists measure Y, one measures X. For each of these experiments they average over many measurements, determine the average value of each one (*e.g.,* for experiment 1, find $\langle \mathrm{X}_1,\mathrm{X}_2,\mathrm{X}_3\rangle$).

(c) Which of these experiments find the same product of measurements for each state that they receive?

(d) Under the hidden variable hypothesis the measurements by different scientists are preordained and the randomness is instead due to hidden variables encoded in each three spin set. Thus, they can be sent spins with hidden variables that give predetermined $(\mathrm{X}_1,\mathrm{Y}_1,\mathrm{X}_2,\mathrm{Y}_2,\mathrm{X}_3,\mathrm{Y}_3)$ and the randomness is due to different samples of spins being sent. Under this hypothesis show that the experimental results of part (c) violate the hidden variable hypothesis. Thus this gives a simple experimental refutation of hidden variables.

Problem 2

Consider two hydrogen atoms separated by $\vec{r}$ where $r = |\vec{r}|$ is much larger than the Bohr radius. The position of proton 1 is at the origin, electron 1 at $\vec{r}_1$, proton 2 at $\vec{r}$ and electron 2 at $\vec{r} + \vec{r}_2$. In the following, take into account only the motion of the electrons.

(a) Write down the Hamiltonian and separate it into two terms $H = H_\circ + \Delta H$ where $H_\circ$ is the sum of the Hamiltonians of the hydrogen atoms 1 and 2 without considering cross-interactions.

(b) Find the leading terms of ΔH in the limit that $r \gg |\vec{r}_1|, |\vec{r}_2|$.

(c) Under perturbation theory, what is the ground state wave function in terms of the ground state wave function of the hydrogen atom $\Psi_\circ(\vec{x})$? Write down the first nonzero correction to the ground state energy. You may leave your answer as a sum over unevaluated matrix elements.

(d) The energy correction you've calculated is the Van der Waals potential. Does this lead to an attractive or repulsive force between the hydrogen atoms? How does the force depend on r?

Problem 3

Consider a 1-D quantum mechanics problem with Hamiltonian

$$H_1\Psi(x) = -\left(\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V_1(x)\right)\Psi(x).$$

Note: Assume V_1 is chosen so that the ground state for H_1 has zero energy.

(a) Show that the Hamiltonian can be written as $H_1 = A^\dagger A$ where $A = \frac{\hbar}{\sqrt{2m}}\frac{d}{dx} + W(x)$ and $A^\dagger = -\frac{\hbar}{\sqrt{2m}}\frac{d}{dx} + W(x)$, and give the relation between $W(x)$ and $V_1(x)$.

(b) Show that given the ground state wave function $\chi_\circ(x)$, one can $i)$ write

$$W(x) = \frac{\hbar}{\sqrt{2m}}\frac{1}{\chi_\circ(x)}\frac{d\chi_\circ(x)}{dx}$$

and that $ii)$ $A\chi_\circ(x) = 0$.

(c) Now consider the Hamiltonian $H_2 = AA^\dagger$ where the two operators are reversed. Show that the eigenvalues for H_1 and H_2 are the same except that there is no zero energy eigenstate for H_2. (**Hint**: Consider an eigenfunction Ψ_1^n with eigenvalue E_1^n under H_1. Show that one can use this to generate an eigenfunction for H_2. Similarly, show that an eigenfunction Ψ_2^n with eigenvalue E_2^n under H_2 can generate an eigenfunction under H_1.)

(d) For the particle in a box where $V_1(x) = \text{constant}$ for $0 < x < L$ and $V_1(x) = \infty$ for $x < 0$, $x > L$, find the Hamiltonian H_2 given your knowledge of the particle in a box solutions.

Problem 4

Assume that Saturn and the Sun can be regarded as sources of blackbody radiation. The radius of Saturn is $R_{saturn} = 58{,}000$ km, and the mean radius of its orbit around the Sun is $r_{saturn} = 1.5 \times 10^{12}$m. From spectroscopic observations, we know that the surface temperature of Saturn is $T_{saturn} = 123$ K. The energy per unit time impinging on Saturn from the Sun is not balanced by the power escaping from Saturn through blackbody radiation. The surface temperature of the Sun is $T_S = 5,800$ K.

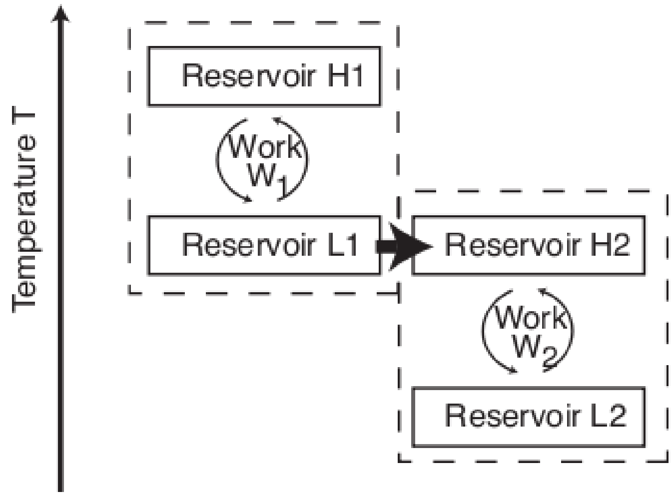
(a) Write down a rate equation for the change of Saturn's total energy U per unit time assuming that it loses energy from its surface only by blackbody radiation and it gains energy only by absorption of blackbody radiation from the Sun. Hint: Stefan's constant is $\sigma = \frac{\pi^2 k_B^4}{60\hbar^3 c^3}$

(b) Compute the value of dU/dt in Watts.

(c) As a possible cause of the nonzero dU/dt, we postulate gravitational contraction: modeling Saturn as a homogeneous sphere of radius R and mass M, its gravitational potential energy is $U_g = -\frac{3}{5}GM^2/R_{saturn}$. The (constant) mass of Saturn is $M = 5.69 \times 10^{26}$ kg, and the gravitational constant is $G = 6.672 \times 10^{-11}$ N-m^2-kg^{-2}. If we assume $dU/dt = dU_g/dt$, what is the rate dR_{saturn}/dt at which the radius of Saturn is decreasing?

Problem 5

In a steam power plant, steam engines work in pairs, the heat output from one being the approximate heat input of the second; see diagram.



The operating temperatures of the first engine are $T_{H1} = 670°$ C and $T_{L1} = 440°$ C, and of the second $T_{H2} = 440°$ C and $T_{L2} = 290°$ C.

(a) If the heat of combustion of coal is 2.8×10^7 J-kg^{-1}, at what rate must coal be burned if the plant is to put out $P_{total} = 1,000$ Mega-Watts of power? Assume the efficiency of the engines is 60% of the Carnot efficiency.

(b) Water is used to cool the power plant. If the water temperature is allowed to increase by no more than 6.0° C, estimate how much water must pass through the plant per hour. To get the specific heat of water, recall the definition of one calorie, $1\ cal = 4,186\ J$.

Problem 6

A particle moving in one dimension in a static fluid satisfies the equation of motion

$$m\dot{v}(t) + \gamma v(t) = f(t) \tag{1}$$

where m and $v(t)$ are the mass and velocity of the particle, respectively, and γ is a damping rate. $f(t)$ is a fluctuating force which, at any given time, has a random value over which we average in an ensemble of many realizations. This ensemble average with the time held fixed is indicated by angle brackets $\langle \ldots \rangle$. We will assume the following properties of f under this average:

$$\langle f(t) \rangle = 0, \qquad \langle f(t)\, f(t') \rangle = c\, \delta(t - t'), \tag{2}$$

where c is some constant and $\delta(\ldots)$ is the Dirac delta function. You will not need to use any further properties of the ensemble average beyond equation (**??**).

(a) Verify that equation (**??**) can be written as

$$\frac{d}{dt}\left(v(t)\, e^{\gamma t/m}\right) = \frac{1}{m} e^{\gamma t/m} f(t)$$

and integrate this equation to find the formal solution for $v(t)$.

(b) Using equation (**??**), derive expressions for $\langle v(t) \rangle$ and $\langle v(t)\, v(t') \rangle$.

(c) Assume that the fluid is in thermal equilibrium at temperature T. Consider the case where the particle equilibrates with the fluid to determine the constant c. Hint: use the equipartition theorem.