Exam #: ______________________

Printed Name: ______________________

Signature: ______________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Unified Graduate Examination

Part I

Analytical Mechanics

Friday, January 5, 2018, 10:00 to 12:40

The examination booklet is numbered in the upper right-hand corner of the cover page. Print and then sign your name in the spaces provided on the cover page. For identification purposes, be sure to submit this page together with your answers when the exam is finished.

There are four questions, each beginning on a new page. Read all four questions before attempting any answer. You may answer as many questions as you wish, however, only your top three scores will be used in the evaluation of your performance for a Ph.D. pass in this area, or your top two scores for a master's pass.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. When you start a new problem, start a new page. Place both the exam number and the question number on all pages you wish to have graded. You are encouraged to use the constants on the following page, where appropriate, to help you solve the problems.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic and will be provided. **Personal calculators are not allowed**. Dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries are not allowed. No other papers or books may be used.**

When you have finished, come to the front of the room. For each problem, put the pages in order and staple them together. Then put all problems in numerical order and place them in the envelope provided. Finally, hand the envelope to the proctor.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg (0.511 MeV/c^2)
Proton rest mass (m_p)	1.673×10^{-27} kg (938 MeV/c^2)
Neutron rest mass (m_n)	1.675×10^{-27} kg (940 MeV/c^2)
Atomic mass unit (AMU)	1.66×10^{-27} kg
Atomic weight of a hydrogen atom	1 AMU
Atomic weight of a nitrogen atom	14 AMU
Atomic weight of an oxygen atom	16 AMU
Planck's constant (h)	6.63×10^{-34} J·s
Bohr Magneton (μ_B)	9.27×10^{-28} J/G
Speed of light in vacuum (c)	3.00×10^{8} m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Gravitational constant (G)	6.67×10^{-11}N·m^2/kg^2
Permeability of free space (μ_0)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space (ϵ_0)	8.85×10^{-12} F/m
Mass of earth (M_E)	5.98×10^{24} kg
Equatorial radius of earth (R_E)	6.38×10^{6} m
Mass of Sun (M_S)	1.99×10^{30} kg
Radius of Sun (R_S)	6.96×10^{8} m
Classical electron radius (r_0)	2.82×10^{-15} m
Density of water	1.0 kg/liter
Density of ice	0.917 kg/liter
Specific heat of water	4180 J/(kg K)
Specific heat of ice	2050 J/(kg K)
Heat of fusion of water	334 kJ/kg
Heat of vaporization of water	2260 kJ/kg
Specific heat of oxygen (c_V)	21.1 J/mole·K
Specific heat of oxygen (c_P)	29.4 J/mole·K
Gravitational acceleration on Earth (g)	9.8 m/s^2
1 atmosphere	1.01×10^{5} Pa

Problem 1.1

A raindrop falls vertically through a cloud and gains mass by absorbing tiny droplets of water from the cloud. Assuming the raindrop is spherical at all times, denote the time-dependent mass, radius, and downward velocity of the raindrop as $m(t)$, $r(t)$, and $v(t)$, respectively.

The mass density of the raindrop is a constant ρ whereas the average mass density of the cloud is a constant σ. The tiny cloud droplets are assumed to be stationary.

(a) [2 points] Express dm/dt in terms of r and v. (Assume the raindrop fully absorbs every droplet it encounters.)

(b) [2 points] Write down Newton's equation for the raindrop (you may ignore air resistance).

(c) [1 point] Write down the relation between dm/dt and dr/dt that results from the raindrop being spherical.

(d) [1 point] Use the equations from parts (a) and (c) to write $v(t)$ in terms of $r(t)$ and/or its derivatives.

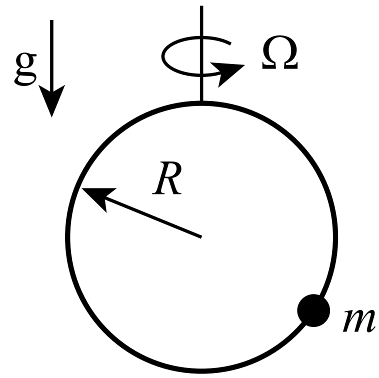
(e) [4 points] Use the preceding results to find a differential equation for $r(t)$ alone. Verify that the ansatz

$$r(t) = Cgt^2 \frac{\sigma}{\rho}$$

with C a dimensionless constant is a possible solution. Calculate C and show that $dv/dt = g/7$ with g the local acceleration due to gravity.

Problem 1.2

A mass m is confined to slide along a massless and frictionless circular ring with radius R. The ring is spun on its axis with an angular velocity Ω.



a) (2 points) Choose an appropriate generalized coordinate and find the Lagrangian for this system.

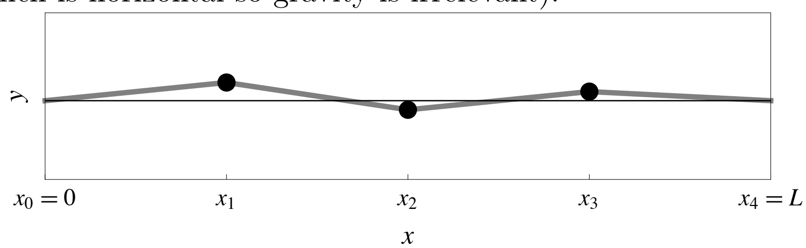
b) (3 points) Derive the equation of motion.

c) (2 points) What are the equilibrium positions of the mass in this system?

d) (3 points) Find the frequencies of small oscillations about these equilibria and any conditions on Ω required for these oscillations.

1.3

Three masses of equal size M each are spaced at equal intervals on a massless rubber band of tension T and total length L. They can deviate from their equilibrium positions (arranged in a straight line) by moving in the xy plane (which is horizontal so gravity is irrelevant).



(a) **(2 points)** How many longitudinal and transverse normal modes does the system have?

(b) **(4 points)** Denoting the transverse position of mass j (with $j = 1, 2, 3$) by y_j, write the equation of motion for y_j in the limit of small deviations from equilibrium. Assume that the tension T is so large that its variation with elongation can be neglected. Hint: It is useful to introduce the variables $y_0 = y_4 = 0$ for the fixed end points.

(c) **(4 points)** Find the (angular) frequencies of all the transverse normal modes.

Problem 1.4

A mass particle m moves in the circular orbit $r = 1/\alpha$ in the potential field

$$U(r) = -C\frac{e^{-\alpha r}}{r}, \tag{1}$$

with $C > 0$ and $\alpha > 0$ constants.

a. (5 points) Find the total energy and the centrifugal potential for the particle in terms of $C > 0$, α, and m.

b. (5 points) Show that the orbit is stable to radial perturbations.