Exam #: ______________________

Printed Name: ______________________

Signature: ______________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Unified Graduate Examination

Part I

Analytical Mechanics

Friday, January 6, 2017, 10:00 to 12:40

The examination booklet is numbered in the upper right-hand corner of the cover page. Print and then sign your name in the spaces provided on the cover page. For identification purposes, be sure to submit this page together with your answers when the exam is finished.

There are four questions, each beginning on a new page. Read all four questions before attempting any answer. You may answer as many questions as you wish, however, only your top three scores will be used in the evaluation of your performance for a Ph.D. pass in this area, or your top two scores for a master's pass. (These scores will be added to your aggregate if taking the exam "under the old rules".)

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. When you start a new problem, start a new page. Place both the exam number and the question number on all pages you wish to have graded. You are encouraged to use the constants on the following page, where appropriate, to help you solve the problems.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic and will be provided. **Personal calculators are not allowed**. Dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries are not allowed. No other papers or books may be used.**

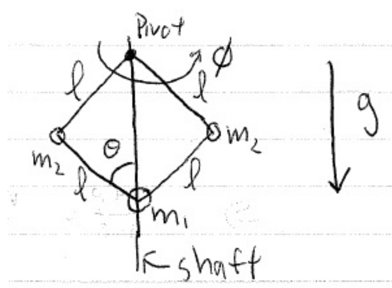
When you have finished, come to the front of the room. For each problem, put the pages in order and staple them together. Then put all problems in numerical order and place them in the envelope provided. Finally, hand the envelope to the proctor.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg (0.511 MeV/c^2)
Proton rest mass (m_p)	1.673×10^{-27} kg (938 MeV/c^2)
Neutron rest mass (m_n)	1.675×10^{-27} kg (940 MeV/c^2)
Atomic mass unit (AMU)	1.66×10^{-27} kg
Atomic weight of a hydrogen atom	1 AMU
Atomic weight of a nitrogen atom	14 AMU
Atomic weight of an oxygen atom	16 AMU
Planck's constant (h)	6.63×10^{-34} J·s
Bohr Magneton (μ_B)	9.27×10^{-28} J/G
Speed of light in vacuum (c)	3.00×10^{8} m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Gravitational constant (G)	6.67×10^{-11} N·m^2/kg^2
Permeability of free space (μ_0)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space (ϵ_0)	8.85×10^{-12} F/m
Mass of earth (M_E)	5.98×10^{24} kg
Equatorial radius of earth (R_E)	6.38×10^{6} m
Mass of Sun (M_S)	1.99×10^{30} kg
Radius of Sun (R_S)	6.96×10^{8} m
Classical electron radius (r_0)	2.82×10^{-15} m
Density of water	1.0 kg/liter
Density of ice	0.917 kg/liter
Specific heat of water	4180 J/(kg K)
Specific heat of ice	2050 J/(kg K)
Heat of fusion of water	334 kJ/kg
Heat of vaporization of water	2260 kJ/kg
Specific heat of oxygen (c_V)	21.1 J/mole·K
Specific heat of oxygen (c_P)	29.4 J/mole·K
Gravitational acceleration on Earth (g)	9.8 m/s^2
1 atmosphere	1.01×10^{5} Pa

Problem 1.1

A bead of mass m_1 slides on a frictionless vertical shaft under gravity, with gravitational acceleration g. It is connected by four massless rigid rods of length ℓ to two other masses of equal mass m_2. (See figure). These masses are each connected by a rigid rod of the same length length l to a fixed pivot mounted on the shaft. The rods are connected in such a way that all four points (the pivot, and the three masses) always lie in a common vertical plane. The entire apparatus is free to rotate about the shaft, and is illustrated in the figure below. The angle θ between the rods and the shaft is also free to change.



The entire system is frictionless.

a) Write down the Lagrangian for this system in terms of the variables θ and ϕ shown in the figure. (3 points)

b) Find two independent conserved quantities. (3 points)

c) If $\frac{d\phi}{dt}(t = 0) \neq 0$ and $\theta(t = 0) \neq 0$, can $\theta(t > 0)$ *ever* $= 0$? If not, why not? (4 points)

Problem 1.2

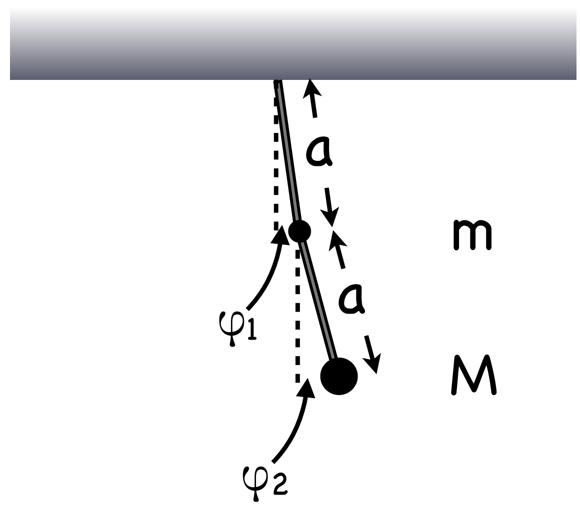
A double pendulum consists of masses m and M attached to two rods of negligible mass separated by a distance a as shown, such that motion occurs only in the plane of the page. The angle of m with respect to the vertical is φ_1, while the angle of M with respect to the vertical is φ_2. Considering only the small amplitude motion:

(a) Write down the Lagrangian of the system (2 points)

(b) Write the equations of motion (2 points)

(c) Calculate the frequencies of the normal modes of the system (3 points)

(d) Sketch the motion that corresponds to the normal modes, and discuss the behavior of the system in the limits $m \ll M$ and $m \gg M$ (3 points)



Problem 1.3

Consider a rocket initially of mass m_o that has a constant fuel burn rate (i.e. mass consumption rate), α, and a constant exhaust velocity u. Assume all velocities are $\ll c$, the speed of light.

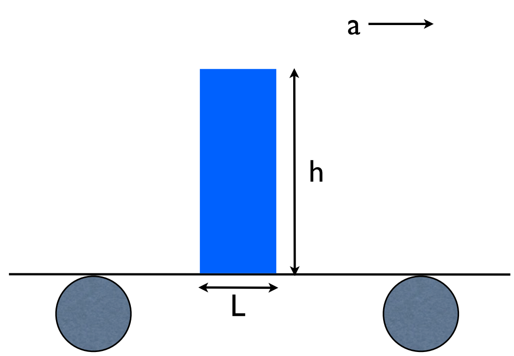
(a) (4 points) If the rocket starts from rest in free space (i.e. $g = 0$) by emitting mass, at what fraction of the initial mass is its momentum maximized?

(b) (2 points) Let g be the magnitude of the gravitational acceleration of Earth, assumed to be constant. If the rocket launches normal to the surface of the Earth, what is the minimum u such that the rocket will lift-off immediately after firing?

(c) (4 points) In the same situation as (b), what is the rocket's velocity in the early stages of the ascent? What minimum fraction, f, of the initial mass must be fuel to reach escape velocity, v_e? Assume that the time to reach escape velocity is $\ll m_o/\alpha$.

Problem 1.4

A truck carries a rectangular block of uniform mass density with height h, a square base with width L, and total mass M. The truck accelerates with constant acceleration a. Assume the box does not slide.



(a) [3 points] In the frame of reference of the truck, draw all forces and pseudo forces acting on the block (and where they act) when the block just starts tipping over.

(b) [4 points] Calculate for what value of a the block starts to tip over.

(c) [2 points] If there is an additional small mass m glued to the center of the top of the block, for what value of a does the block start to tip over now?

(d) [1 point] Verify that your answer to (c) is smaller than what you found in part (b).