Exam #: ______________________

Printed Name: ______________________

Signature: ______________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Unified Graduate Examination

Part I

Analytical Mechanics

Tuesday, April 2, 2018, 10:00 to 12:40

The examination booklet is numbered in the upper right-hand corner of the cover page. Print and then sign your name in the spaces provided on the cover page. For identification purposes, be sure to submit this page together with your answers when the exam is finished.

There are four questions, each beginning on a new page. Read all four questions before attempting any answer. You may answer as many questions as you wish, however, only your top three scores will be used in the evaluation of your performance for a Ph.D. pass in this area, or your top two scores for a master's pass.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. When you start a new problem, start a new page. Place both the exam number and the question number on all pages you wish to have graded. You are encouraged to use the constants on the following page, where appropriate, to help you solve the problems.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic and will be provided. **Personal calculators are not allowed**. Dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries are not allowed. No other papers or books may be used.**

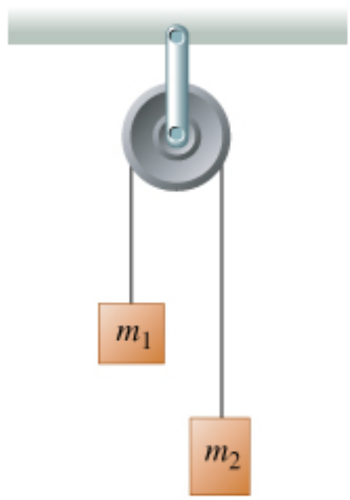
When you have finished, come to the front of the room. For each problem, put the pages in order and staple them together. Then put all problems in numerical order and place them in the envelope provided. Finally, hand the envelope to the proctor.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg (0.511 MeV/c^2)
Proton rest mass (m_p)	1.673×10^{-27} kg (938 MeV/c^2)
Neutron rest mass (m_n)	1.675×10^{-27} kg (940 MeV/c^2)
Atomic mass unit (AMU)	1.66×10^{-27} kg
Atomic weight of a hydrogen atom	1 AMU
Atomic weight of a nitrogen atom	14 AMU
Atomic weight of an oxygen atom	16 AMU
Planck's constant (h)	6.63×10^{-34} J·s
Bohr Magneton (μ_B)	9.27×10^{-28} J/G
Speed of light in vacuum (c)	3.00×10^{8} m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Gravitational constant (G)	6.67×10^{-11}N·m^2/kg^2
Permeability of free space (μ_0)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space (ϵ_0)	8.85×10^{-12} F/m
Mass of earth (M_E)	5.98×10^{24} kg
Equatorial radius of earth (R_E)	6.38×10^{6} m
Mass of Sun (M_S)	1.99×10^{30} kg
Radius of Sun (R_S)	6.96×10^{8} m
Classical electron radius (r_0)	2.82×10^{-15} m
Density of water	1.0 kg/liter
Density of ice	0.917 kg/liter
Specific heat of water	4180 J/(kg K)
Specific heat of ice	2050 J/(kg K)
Heat of fusion of water	334 kJ/kg
Heat of vaporization of water	2260 kJ/kg
Specific heat of oxygen (c_V)	21.1 J/mole·K
Specific heat of oxygen (c_P)	29.4 J/mole·K
Gravitational acceleration on Earth (g)	9.8 m/s^2
1 atmosphere	1.01×10^{5} Pa

Problem 1.1

In a simple Atwood machine, two masses are suspended, under constant gravity, from the ends of a flexible, massless, inextensible rope that passes over a inertialessly rotating pulley. Here, the two masses are m_1 and m_2, the rope is of length $l_\circ$, and the constant gravitational acceleration g is downward.



Suppose that the second mass is replaced by a live monkey of equal mass that climbs up the rope at speed $v(t)$ relative to the rope. Treating the monkey's motion as a (time-dependent) constraint in the Lagrangian formalism, answer the following questions:

a. (5 points) Find the acceleration of the mass, m_1, if the monkey climbs up the rope with uniform speed $v_\circ$.

b. (4 points) Find the acceleration of the mass m_1, if the monkey climbs up the rope with nonuniform speed $v(t)$ but constant acceleration, $\dot{v}(t) = a_\circ$.

c. (1 point) Is it possible for the monkey to lift a weight greater than its own if it climbs vigorously enough? If so, under what condition?

Problem 1.2

A particle of mass m moves without friction in one dimension along the x axis in a potential $U(x) = \frac{1}{2}k|x|^\nu$.

a) If the particle starts at rest at $x = A$, show that its period of oscillation T is given by

$$T = C\left(\frac{m}{k}\right)^\alpha A^\beta \ , \tag{1}$$

where C is a dimensionless constant that depends only on ν, and give expressions for C, α and β. Your expression for C may be an integral which you need not evaluate; your expressions for α and β must be explicit. (7 points)

b) For what values of ν does the period grow as A is increased? For what values does it get smaller as A is increased? Are there any values of ν for which the period is independent of A?

(3 points)

Problem 1.3

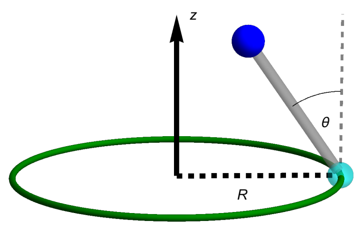
A particle of mass m is subject to an attractive central force $\mathbf{f}_1(r) = \hat{\mathbf{r}} f(r)$ and a frictional force $\mathbf{f}_2 = -\lambda \mathbf{v}$ where $\mathbf{v}$ is the velocity of the particle with $\lambda > 0$. The particle initially has an angular momentum $\mathbf{J}_o$ about $r = 0$.

(a) (4 points) Write out the equations of motion of the particle in a suitable coordinate system.

(b) (3 points) Derive a differential equation that governs the angular momentum of the system as a function of time. Solve this differential equation.

(c) (3 points) What is the rate of dissipation of energy in this system in terms of the parameters and time derivatives of the generalized coordinates?

Problem 1.4



A point mass m is mounted at the top of a massless rigid rod of length $L > 0$, whose other end is connected to a circular ring by a frictionless hinge. The rod can pivot around the hinge in the plane containing the z axis, which points up (see figure). The circle has radius $R > L$ and is rotating around the z axis at constant angular velocity $\omega > 0$. The mass is subject to the downward acceleration of gravity, g. Use the angle θ between the rod and the vertical to describe the position of the point mass.

(a) **(4 points)** Find the Lagrangian for the coordinate $\theta(t)$.

(b) **(3 points)** In terms of R, L and θ, find the (positive) angular frequency ω for which the angle θ remains constant during the rotation.

(c) **(3 points)** If the mass is at the equilibrium angle θ but has a generalized velocity $\dot{\theta} \neq 0$, what is the z component of the torque that must be exerted on the rotating ring to maintain the angular velocity ω obtained in **(b)**?