Exam #: ______________________

Printed Name: ______________________

Signature: ______________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Unified Graduate Examination

Part I

Analytical Mechanics

Tuesday, April 4, 2017, 10:00 to 12:40

The examination booklet is numbered in the upper right-hand corner of the cover page. Print and then sign your name in the spaces provided on the cover page. For identification purposes, be sure to submit this page together with your answers when the exam is finished.

There are four questions, each beginning on a new page. Read all four questions before attempting any answer. You may answer as many questions as you wish, however, only your top three scores will be used in the evaluation of your performance for a Ph.D. pass in this area, or your top two scores for a master's pass.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. When you start a new problem, start a new page. Place both the exam number and the question number on all pages you wish to have graded. You are encouraged to use the constants on the following page, where appropriate, to help you solve the problems.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic and will be provided. **Personal calculators are not allowed**. Dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries are not allowed. No other papers or books may be used.**

When you have finished, come to the front of the room. For each problem, put the pages in order and staple them together. Then put all problems in numerical order and place them in the envelope provided. Finally, hand the envelope to the proctor.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg ($0.511\,\mathrm{MeV/c^2}$)
Proton rest mass (m_p)	1.673×10^{-27} kg ($938\,\mathrm{MeV/c^2}$)
Neutron rest mass (m_n)	1.675×10^{-27} kg ($940\,\mathrm{MeV/c^2}$)
Atomic mass unit (AMU)	1.66×10^{-27} kg
Atomic weight of a hydrogen atom	1 AMU
Atomic weight of a nitrogen atom	14 AMU
Atomic weight of an oxygen atom	16 AMU
Planck's constant (h)	6.63×10^{-34} J·s
Bohr Magneton (μ_B)	9.27×10^{-28} J/G
Speed of light in vacuum (c)	3.00×10^{8} m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Gravitational constant (G)	$6.67 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2}$
Permeability of free space (μ_0)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space (ϵ_0)	8.85×10^{-12} F/m
Mass of earth (M_E)	5.98×10^{24} kg
Equatorial radius of earth (R_E)	6.38×10^{6} m
Mass of Sun (M_S)	1.99×10^{30} kg
Radius of Sun (R_S)	6.96×10^{8} m
Classical electron radius (r_0)	2.82×10^{-15} m
Density of water	1.0 kg/liter
Density of ice	0.917 kg/liter
Specific heat of water	4180 J/(kg K)
Specific heat of ice	2050 J/(kg K)
Heat of fusion of water	334 kJ/kg
Heat of vaporization of water	2260 kJ/kg
Specific heat of oxygen (c_V)	21.1 J/mole·K
Specific heat of oxygen (c_P)	29.4 J/mole·K
Gravitational acceleration on Earth (g)	$9.8\,\mathrm{m/s^2}$
1 atmosphere	1.01×10^{5} Pa

Pauli spin matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \ , \ \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \ , \ \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \ .$$

Stirling's formula

$$\log(N!) \approx N \log N - N$$

Integrals

$$\int_{-\infty}^{\infty} e^{-ax^2} \mathrm{d}x = \left(\frac{\pi}{a}\right)^{1/2}$$

$$\int_{-\infty}^{\infty} x^2 \ e^{-ax^2} \mathrm{d}x = \frac{1}{2a} \left(\frac{\pi}{a}\right)^{1/2}$$

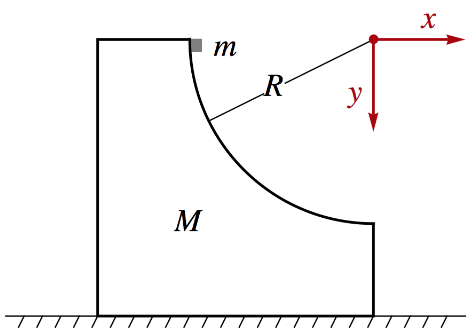
Differential Operators

$$\nabla^2_{\text{spherical}} = \left[\frac{1}{r^2}\partial_r(r^2\partial_r) + \frac{1}{r^2 \sin\theta}\partial_\theta(\sin\theta\partial_\theta) + \frac{1}{r^2\sin^2\theta}\partial^2_\phi\right]$$

$$\nabla^2_{\text{cylindrical}} = \left[\frac{1}{r}\partial_r(r\partial_r) + \frac{1}{r^2}\partial^2_\phi + \partial^2_z\right]$$

1.1

A small cube of mass m slides frictionlessly down a circular path of radius R cut into a large block of mass M, as shown below:



The block M can slide horizontally without friction, and the cube m is subject to a constant downward gravitational acceleration g.

At $t = 0$, the center of the circular arc has horizontal position $X = 0$ in the lab frame; both blocks are at rest, and the small cube m is released at the top of the path. The lab-frame position of the cube is (x, y) where y is the vertical distance below the launch point.

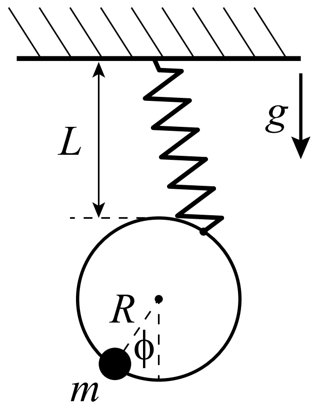
(a) **(3 points)** Write the total energy of the system in terms of the lab-frame coordinate X of the arc's center, and the position (x, y) of the cube.

(b) **(6 points)** Find the velocity of the cube as it leaves the block at $y = R$.

(c) **(1 point)** What is limit of the result in **(b)** when $M \gg m$?

Problem 1.2

A point mass m under the influence of gravity is attached to a massless rotating ring of radius R whose position is characterized by the angle ϕ as shown below. The center of the ring is fixed and the only degree of freedom is rotation about its center. A massless spring with spring constant k is attached to the top of the ring and to a wall a distance L directly above the top of the ring. The spring has an equilibrium length of zero.



a) (5 points) What is the full equation of motion in this system for $\phi(t)$?

b) (3 points) If $\phi(0) = 0$, what is the minimum $\dot{\phi}(0)$ for the ring to spin continuously, that is for $\phi(t)$ to increase monotonically with time?

c) (2 points) What is the frequency of small oscillations in this system?

Problem 1.3

Suppose a particle of mass m is scattered by a target particle of mass $M \gg m$ due to a repulsive force $F = \frac{\gamma}{r^2}$ as shown in the figure. The position of the particle at time t can be labeled by its angle $\psi(t)$ measured relative to vector $\mathbf{u}$. ($\mathbf{u}$ is defined as a vector that points in the direction of the scattering particle at its closest approach as shown in the figure.) You can assume the target particle M is held fixed.

a) What is the scattering angle θ in terms of $\psi_0 \equiv \psi(t)|_{t\to\infty}$?

[1 point]

b) Using kinematics, what is the change in the magnitude of the momentum of the particle m in terms of the initial momentum $p \equiv |\mathbf{p}|$ and scattering angle θ?

[1 point]

c) What is the magnitude of the angular momentum of the particle expressed in terms of $\dot{\psi}$? Is it conserved?

[2 points]

d) Using the repulsive force, re-calculate the change in magnitude of the momentum in terms of the impact parameter b and θ. [Hint: it is helpful to change variables from time to $\psi(t)$.]

[3 points]

e) Using your results from b) and d), calculate the impact parameter b in terms of θ.

[1 point]

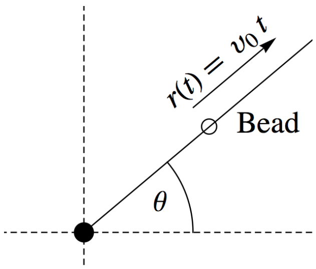
f) Calculate the differential cross section $d\sigma/d\Omega$ where $d\sigma = 2\pi b\, db$ and $d\Omega \equiv 2\pi \sin\theta\, d\theta$.

[2 points]

1.4

A bead of mass m moves along a massless, straight and rigid wire pivoted at the origin with no influence of gravity, as shown in the diagram. The wire is allowed to rotate freely in the xy plane and the bead moves along the wire at a constant speed v_0.

(a) (5 points) Construct an expression for the Lagrangian of this system as a function of the polar angle θ.



(b) (2 points) Derive the equation of motion for the angle $\theta(t)$ from this Lagrangian.

(c) (3 points) Solve the equation of motion for $\theta(t)$ given the initial conditions $\theta(t_0) = 0$, $\dot{\theta}(t_0) = \omega_0$. Assume $t \geq t_0 > 0$.