Exam #: ______________________

Printed Name: ______________________

Signature: ______________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Unified Graduate Examination

Part I

Analytical Mechanics

Monday, March 28, 2016, 10:00 to 12:40

The examination booklet is numbered in the upper right-hand corner of the cover page. Print and then sign your name in the spaces provided on the cover page. For identification purposes, be sure to submit this page together with your answers when the exam is finished.

There are four questions, each beginning on a new page. Read all four questions before attempting any answer. You may answer as many questions as you wish, however, only your top three scores will be used in the evaluation of your performance for a Ph.D. pass in this area, or your top two scores for a master's pass. (These scores will be added to your aggregate if taking the exam "under the old rules".)

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. When you start a new problem, start a new page. Place both the exam number and the question number on all pages you wish to have graded. You are encouraged to use the constants on the following page, where appropriate, to help you solve the problems.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic and will be provided. **Personal calculators are not allowed**. Dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries are not allowed. No other papers or books may be used.**

When you have finished, come to the front of the room. For each problem, put the pages in order and staple them together. Then put all problems in numerical order and place them in the envelope provided. Finally, hand the envelope to the proctor.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg (0.511 MeV/c^2)
Proton rest mass (m_p)	1.673×10^{-27} kg (938 MeV/c^2)
Neutron rest mass (m_n)	1.675×10^{-27} kg (940 MeV/c^2)
Atomic mass unit (AMU)	1.66×10^{-27} kg
Atomic weight of a hydrogen atom	1 AMU
Atomic weight of a nitrogen atom	14 AMU
Atomic weight of an oxygen atom	16 AMU
Planck's constant (h)	6.63×10^{-34} J·s
Bohr Magneton (μ_B)	9.27×10^{-28} J/G
Speed of light in vacuum (c)	3.00×10^{8} m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Gravitational constant (G)	6.67×10^{-11} N·m^2/kg^2
Permeability of free space (μ_0)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space (ϵ_0)	8.85×10^{-12} F/m
Mass of earth (M_E)	5.98×10^{24} kg
Equatorial radius of earth (R_E)	6.38×10^{6} m
Mass of Sun (M_S)	1.99×10^{30} kg
Radius of Sun (R_S)	6.96×10^{8} m
Classical electron radius (r_0)	2.82×10^{-15} m
Density of water	1.0 kg/liter
Density of ice	0.917 kg/liter
Specific heat of water	4180 J/(kg K)
Specific heat of ice	2050 J/(kg K)
Heat of fusion of water	334 kJ/kg
Heat of vaporization of water	2260 kJ/kg
Specific heat of oxygen (c_V)	21.1 J/mole·K
Specific heat of oxygen (c_P)	29.4 J/mole·K
Gravitational acceleration on Earth (g)	9.8 m/s^2
1 atmosphere	1.01×10^{5} Pa

Problem 1

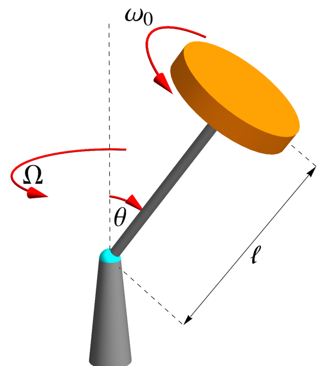
Consider a particle of mass m attached to a spring with restoring force $F_r = -kx$. The particle moves in a horizontal viscous medium which provides a damping force proportional to the particle's velocity, $F_v = -bv$, with v the velocity in the x direction. For this problem the motion is restricted to one dimension.

(a) (2 points) Write the differential equation that describes the motion of the particle.

(b) (5 points) Solve for the motion of the particle.

(c) (3 points) Under what conditions (that is, for what values of the constants, k, b, and m), will the motion be oscillatory?

Problem 2



A gyroscope of mass m and moment of inertia I around its perpendicular axis rotates with angular velocity ω_0 around a massless axle of length ℓ. The axle is pivoted at its end without friction, and gravity acts with acceleration g.

Assume that the resulting precession is completely uniform so that nutation effects are absent, and the angular velocity Ω of the precession is small, $\Omega \ll \omega_0$ (see figure), so that its contribution to the gyroscope's angular momentum is insignificant.

(a) **(3 points)** If $\boldsymbol{L}$ is the dominant (recall $\Omega \ll \omega_0$) angular momentum of the gyroscope with respect to the pivot, and the axle of the gyroscope is parallel to the unit vector

$$\boldsymbol{n}(t) \equiv \begin{pmatrix} \cos \Omega t \sin \theta \\ \sin \Omega t \sin \theta \\ \cos \theta \end{pmatrix},$$

write down a vectorial expression for its time rate of change,

$$\frac{d\boldsymbol{L}}{dt}.$$

Here, θ measures the tilt between the axle and the vertical.

(b) **(2 points)** State an equation relating $\frac{d\boldsymbol{L}}{dt}$ to the torque on the gyroscope, and calculate the torque.

(c) **(5 points)** Express the precession speed Ω in terms of ω_0, ℓ and the parameters m, I of the gyroscope.

Problem 3

A planet of radius R and uniform mass density ρ has a small straight hole drilled from the surface, through the center, to the other side, in which a mass m can move frictionlessly.

(a) (4 points) What is the potential energy $V(r)$ for the mass as a function of separation from the planet's center at $r = 0$? Assume that the planet is much more massive than m and that the hole diameter is very small compared to R.

(b) (3 points) If the mass is dropped from rest into the hole from a height $h = 0$ above the planet's surface, what is the period of the oscillation?

(c) (3 points) If the ball is dropped from rest into the hole from a height $h > 0$ above the planet's surface, write an expression for the period of one full trip. Your expression may contain integrals involving $V(r)$.

Problem 4

A particle with mass m moves in the x-y plane under the influence of a central force, $\vec{F}(\rho) = -f(\rho)\vec{e}_\rho$, where $f(\rho) > 0$, $\rho = \sqrt{x^2 + y^2}$ is the distance to the origin, and $\vec{e}_\rho$ is the radial unit vector.

(a) (2 pts) Write down the Lagrangian, using polar coordinates ρ, ϕ. The Lagrangian may contain integrals or derivatives involving $f(\rho)$ that need not be evaluated explicitly.

(b) (2 pts) What is the momentum p_ϕ conjugate to ϕ and what is its physical meaning?

(c) (6 pts) Write down the Lagrange equations of motion. Eliminate p_ϕ by using the equation of motion for ϕ, to get one equation for the radial motion. What is the effective force governing the radial motion?