Exam #: ______________________

Printed Name: ______________________

Signature: ______________________

PHYSICS DEPARTMENT

UNIVERSITY OF OREGON

Unified Graduate Examination

PART I

Monday, March 30, 2015, 13:00 to 17:00

The examination papers are numbered in the upper right-hand corner of each page. Print and then sign your name in the spaces provided on this page. For identification purposes, be sure to submit this page together with your answers when the exam is finished. Be sure to place both the exam number and the question number on any additional pages you wish to have graded.

You are encouraged to use the constants on the following page, where appropriate, to help you solve the problems.

There are six equally weighted questions, each beginning on a new page. Read all six questions before attempting any answers.

Begin each answer on the same page as the question, but continue on additional blank pages if necessary. Write only on one side of each page. Each page should contain work related to only one problem. When you start a new problem, start a new page.

If you need to leave your seat, wait until everyone else is seated before approaching the proctor.

Calculators may be used only for arithmetic and will be provided. **Personal calculators are not allowed**. Dictionaries may be used if they have been approved by the proctor before the examination begins. **Electronic dictionaries are not allowed. No other papers or books may be used.**

When you have finished, come to the front of the room. For each problem, put the pages in order and staple them together. Then put all problems in numerical order and place them in the envelope provided. Finally, hand the envelope to the proctor.

Please make sure you follow all instructions carefully. If you fail to follow instructions, or to hand in your exam paper on time, an appropriate number of points may be subtracted from your final score.

Constants

Electron charge (e)	1.60×10^{-19} C
Electron rest mass (m_e)	9.11×10^{-31} kg ($0.511\,\mathrm{MeV/c^2}$)
Proton rest mass (m_p)	1.673×10^{-27} kg ($938\,\mathrm{MeV/c^2}$)
Neutron rest mass (m_n)	1.675×10^{-27} kg ($940\,\mathrm{MeV/c^2}$)
Atomic mass unit (AMU)	1.66×10^{-27} kg
Atomic weight of a hydrogen atom	1 AMU
Atomic weight of a nitrogen atom	14 AMU
Atomic weight of an oxygen atom	16 AMU
Planck's constant (h)	6.63×10^{-34} J·s
Speed of light in vacuum (c)	3.00×10^{8} m/s
Boltzmann's constant (k_B)	1.38×10^{-23} J/K
Gravitational constant (G)	$6.67 \times 10^{-11}\mathrm{N\cdot m^2/kg^2}$
Permeability of free space (μ_0)	$4\pi \times 10^{-7}$ H/m
Permittivity of free space (ϵ_0)	8.85×10^{-12} F/m
Classical electron radius (r_0)	2.82×10^{-15} m
Density of water	1.0 kg/liter
Density of ice	0.917 kg/liter
Specific heat of water	4180 J/(kg K)
Specific heat of ice	2050 J/(kg K)
Heat of fusion of water	334 kJ/kg
Heat of vaporization of water	2260 kJ/kg
Specific heat of oxygen (c_V)	21.1 J/mole·K
Specific heat of oxygen (c_P)	29.4 J/mole·K
Gravitational acceleration on Earth (g)	$9.8\,\mathrm{m/s^2}$
1 atmosphere	1.01×10^{5} Pa

Problem 1

Stars in a galaxy move in the gravitational potential of a spherically symmetric "halo" of dark matter whose density distribution is such that stars moving in circular orbits about the center of the galaxy, all move at the same speed v_c.

a. (1 point) What is the magnitude of the gravitational force towards the galactic center on a star of mass m a distance r from the center?

b. (1 point) What is the gravitational potential felt by such a star?

c. (2 points) What is the effective radial potential experienced by such a star, if it has angular momentum L about the galactic center?

d. (4 points) What is the frequency, ω_r, of small radial oscillations of such a star about a circular orbit of radius $r_\circ$?

e. (2 points) A star is initially moving perpendicular to the line from itself to the galactic center at speed $v_\circ = v_c(1-\epsilon)$, $\epsilon \ll 1$ at a distance $r_\circ$ from the galactic center. To lowest order in ϵ, what is the angle between a line drawn from the center of the galaxy to the star's initial location, and one drawn to the next point at which the star is again a distance $r_\circ$ from the galactic center?

Problem 2

A car, starting from rest, travels a distance D, comes to rest, and then returns to its starting point. The car does not come to a stop when it returns to its starting point. Assume all travel is along a straight line with maximum acceleration α, and maximum (braking) deceleration β.

a. (7 points) What is the minimum time required for the car to travel the distance D?

b. (3 points) What is the minimum time required for the car to return to its starting point from distance D?

Problem 3

A point-like particle with mass m moves on a parabolic surface whose height above the x-y plane is given by

$$\mathrm{h}(x, y) = \mathrm{k}(x^2 + y^2), \tag{1}$$

where k is a positive constant. The gravitational acceleration is uniform and given by $\mathbf{g} = -\mathrm{g}\hat{\mathbf{z}}$, where g is a positive constant. Ignore friction, and rotation of the particle for this problem.

a. (4 points) Write the Lagrangian for the particle.

b. (3 points) From the Lagrangian, find the equation-of-motion for the particle.

c. (3 points) If the particle is constrained to move only in two dimensions, the x and z directions, what restrictions, if any, are required for the motion to be well-described as simple harmonic motion?

Problem 4

Two ships are communicating at sea with omni-directional radio antennae mounted 20 meters above the water surface (we ignore polarization and consider the water surface flat). The frequency of the transmitters are 300 MHz and their output powers are 100 watts. As they approach one another, the signal received by one from the other goes through a succession of maxima and minima.

a. (2 points) Explain this phenomenon with a drawing. (hint: consider the direct line-of-sight signal, and one reflected from the water surface.)

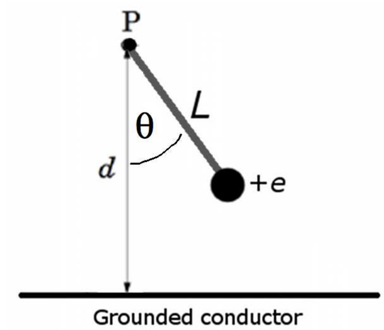
b. (3 points) Find the separation of the two ships at the first minimum, as the ships approach from a large distance.

c. (3 points) What would be the intensity of the signal at the distance of the first minimum in the absence of the reflected wave?

d. (2 points) Suppose the receiver sensitivity is 10^{-12} W/m^2. What is the rms magnetic field strength of the signal at the limit of sensitivity, at a large distance?

Problem 5

Consider a particle of mass m and charge e connected to a fixed point P by a massless rod of length L. Neglect gravity and magnetic forces. There is an infinite grounded conducting plane at a distance d from P (see the Figure). Denote the angle that the rod makes with the vertical as θ.

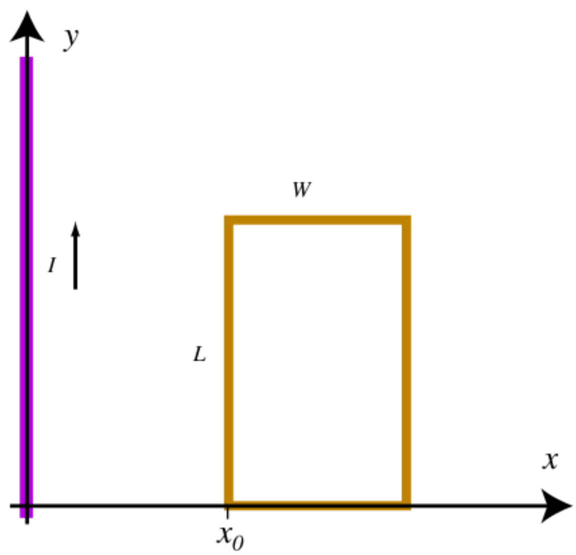


a. (5 points) What is the work required to displace the charge from $\theta = 0$ to $\theta = \pi/2$?

b. (5 points) Compute the frequency of oscillations if θ is small.

Problem 6

Consider a rectangular wire loop of length L and width W moving parallel to the x axis at a constant speed of v. The loop lies in the x-y plane.



An inhomogeneous magnetic field is generated by a thin straight wire along the y axis, carrying a current I. The initial goal is to calculate the induced electromotive force $\mathcal{E}$ in the loop at the instant when it passes position x_0, see the figure. The loop and the straight wire are all in the x-y plane.

a. (3 points) Calculate the magnetic flux Φ_B through the loop as a function of time. To define the sign, assume that positive flux corresponds to magnetic fields pointing in the positive z direction.

b. (3 points) Determine the electromotive force $\mathcal{E}$ from the result of a.

c. (2 points) If the Ohmic resistivity of the loop is ρ and the cross section of the wire is A, what is the induced current I_{loop} in terms of the electromotive force? Is the current clockwise or counter-clockwise?

d. (2 points) Calculate the force required to keep the wire loop moving at the given speed v in the x-direction. (You don't have to calculate the constraint forces holding the motion of the loop parallel to the x axis.)